Viser Technology Pte. Ltd.
Mechanical Engineering Science
DOI: 10.33142/mes.v3i1.4330

Research Article Open Access

Mechanical Analysis for Two-layered Ultrahigh Pressure Apparatus with Interlayer Pressure

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Abstract:
To improve the bearing pressure capacity of ultrahigh pressure apparatus, the internal autofrettaged cylinder with interlayer pressure (ACCIP) is introduced, and the analytical model for the ACCIP structure and its derivation are presented as well. Calculation showed that the ACCIP method enhance bearing pressure of the apparatus obviously; optimization results revealed that under the working pressure p_w=1.07σ_s and different radius ratios, the ACCIP method can keep the apparatus in deformed-elastically state; And when the p_w=1.07σ_s , the minimum radius ratio was approximate 3.29, in this case, no yielding happened. The above results demonstrate that the ACCIP method is a promising technique to improve the bearing pressure of ultrahigh pressure apparatus, and the analytical model for the ACCIP method is also reasonable. In addition, the minimum radius ratio ro/ri under randomly specified workload can be **Fig.**d out by the analytical model proposed in this work.

Keywords: *Mechanical analysis; Ultrahigh pressure; Interlayer pressure; analytical mode*

1 Introduction

Ultrahigh pressure apparatus has a widespread application prospect in nuclear engineering, powder metallurgy, petrochemical, synthetic quartz, artificial diamond, and jet cutting technology. And bearing capacity is one of the focus problem in ultrahigh pressure apparatus. Wang, Yuan, Hu, Wang, & He (2007) built a theoretical model for shrinkage vessel and pointed out that the maximum bearing capacity of single-layer cylinders was just close to half of the yield stress; however, that of shrink-fit vessel can almost close to the yield strength. Miraje, & Patil (2011) calculated shrinkage pressures between two contacting cylinders to reduce the hoop stress and to make it more uniform over the thickness. However, Yuan & Liu (2012) revealed that the shrinkage pressures would result in reverse yielding by the method of calculation and simulation. Considering medium principal stress and brittle softening, Zhu, Zhao, Zhang, Zhang, Wang (2015), and Shufen, Mahanta and Dixit (2019) presented the elastic-brittle-plastic unified solutions of limit internal pressure for double-layered combined thick-walled cylinder. As to autofrettaged apparatus, Zhu & Yang (1998) pointed out that if its thickness was increased infinitely, an autofrettaged cylinder had limitless strength , but a larger pressure need be applied on the cylinder before its use. The study performed in Lee, Lee, Yang, Kim, Cha, & Hong (2009) revealed that due to Bauschinger effect the compressive residual stress of the strain-hardening model for the autofrettaged apparatus is lower than that of the elastic-perfectly plastic model; based on the above study, a Matlab-based GUI for the autofrettaged apparatus was developed by Yang, Lee, Lee, Kim, Cha, & Hong, (2009). The results also showed that the magnitudes of calculated hoop residual stress at the internal radius of the vessel strongly depended on the employed material model (Maleki, Farrahi, Jahromi, & Hosseinian, 2010); Ruilin Zhu, Guolin Zhu, & Aifeng Mao found out the safe and optimum workload conditions for autofrettaged cylinders by a set of simplified equations. And the hoop residual stress of the autofrettaged apparatus under different conditions, was analyzed by analytical and FE analysis (Shim, Kim, Cha, & Hong, 2010; Alexandrov, 2020). Wahi, Ayob, & Elbasheer (2011) proposed an analytical procedure of thick-walled cylinder under the aim of predicting the required autofrettaged pressure for various levels of allowable pressure and attaining optimum fatigue life. Further studies indicated that under specified geometric dimensions and materials, optimum autofrettaged pressure approached to 1.5 times the working pressure, but Bauschinger effect resulted in earlier onset of

Received 25 February 2021; Accepted 28 May 2021

reverse yielding (Hu & Puttagunta, 2012). Literatures (Zhu, Zhu, & Tang, 2012; Li, 2021) demonstrated the maximum bearing capacity of an autofrettaged cylinder was approximately equal to yielding strength of the materials on the basis of the ideal conditions.

To increase elastic-limit capacity and obtain the optimum radial dimension of the ultrahigh pressure apparatus, the autofrettaged apparatus with interlayer pressure (ACCIP) was set up and an analytical model was also presented in this work. The present work was lay out as follows: firstly, derivation of analytical model and structural optimum model was presented on the basis of the ideal conditions in literature (Zhu, Zhu, & Tang, 2012); next, the total radius ratios were presented at the limit workload of the $p_w = 1.07\sigma_s$; afterwards, the stresses field on the ACCIP method under the different total radius ratios and total radius ratio were analyzed. Results demonstrated that the limit workload were close to 1.5 times of the yield strength σ_s by the using ACCIP method, besides the optimum total radius ratio was approximately equal to 3.29 under the limit workload of the 1.07 σ_s.

2 Theoretical model

2.1 Geometric model

The ultrahigh pressure apparatus with ACCIP method consisted of internal cylinder, the external cylinder, and interlayer medium liquid, which was shown schematically in Fig. 1, where the Δ is near to zero.

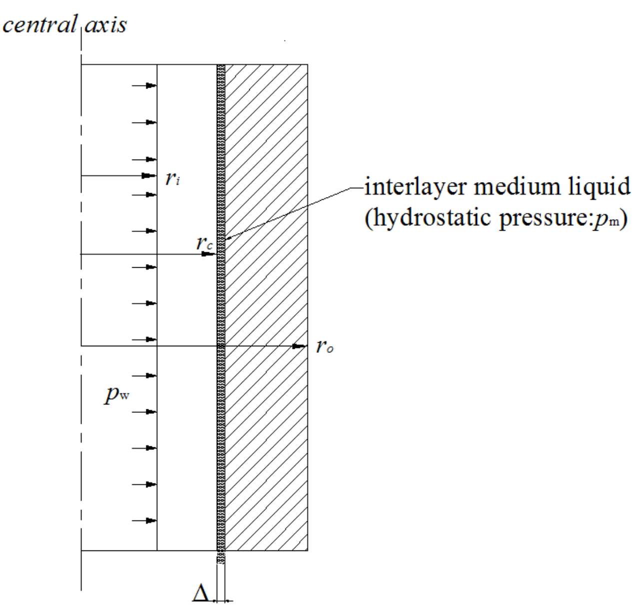


Fig.1. Schematic view

2.2 Mathematical model

Since it was autofrettaged before use, the autofrettaged cylinder underwent the interlayer workload of the pressure p_m, as well as residual stresses. The resultant stresses were listed as follows:

$$\sigma_r^{total} = \sigma_r + \sigma_r^r \tag{1a}$$

$$\sigma_\theta^{total} = \sigma_\theta + \sigma_\theta^r \tag{1b}$$

where σ_r^{total}, σ_θ^{total} are the resultant radial, hoop stresses; σ_r^r, σ_θ^r are the residual radial, hoop stresses; and σ_r, σ_θ are the radial, hoop stresses, respectively.

Furthermore, according to Lame's solution (Miraje, & Patil, 2011), stress components of the internal cylinder were derived as below:

$$\sigma_r = \frac{p_w r_i^2 - p_m r_c^2}{r_c^2 - r_i^2} - \frac{(p_w - p_m) r_i^2 r_c^2}{(r_c^2 - r_i^2) r^2} \tag{2a}$$

$$\sigma_\theta = \frac{p_w r_i^2 - p_m r_c^2}{r_c^2 - r_i^2} + \frac{(p_w - p_m) r_i^2 r_c^2}{(r_c^2 - r_i^2) r^2} \tag{2b}$$

where r_i was internal radius, r_c was the radius of interlayer medium liquid referring to Fig. 1, r was radius variable, p_m was the interlayer pressure, and p_w was the workload when the internal surface of the internal cylinder was at elastic limit state.

When internal cylinder was autofrettaged, the residual stresses in plastic region were as followings:

$$\sigma_r^r = \sigma_s \ln(r/r_c) - p_p(1 - r_c^2/r^2)/[1-(1/K_i)^2] \tag{3a}$$

$$\sigma_\theta^r = \sigma_s[1+\ln(r/r_c)] - p_p(1 + r_c^2/r^2)/[1-(1/K_i)^2] \tag{3b}$$

where the K_i was equal to r_c/r_i.

And the residual stresses in elastic region were as following:

$$\sigma_r^r = [(\sigma_s r_m^2/2r_c^2) - p_p/(K_c^2-1)](1 - r_c^2/r^2) \tag{4a}$$

$$\sigma_\theta^r = [(\sigma_s r_m^2/2r_c^2) - p_p/(K_c^2-1)](1 + r_c^2/r^2) \tag{4b}$$

where the r_m was the radius of elastic-plastic interface in the internal cylinder, p_p was the corresponding pressure on internal wall of internal cylinder when the radius of elastic-plastic interface in the internal cylinder was r_m and the K_c was equal to r_c/r_c.

On the basis of the third strength theory (Borei, Sidebottom, Seely, & Smith, 1978) the equivalent stress was expressed as follows:

$$\sigma_{eq}^{III} = \sigma_\theta - \sigma_r = 2\tau_{max} \le \sigma_s \tag{5}$$

where σ_{eq}^{III} is the equivalent stress, τ_{max} is the maximum shear stress, and the σ_{eq}^{III} is equal to the yield strength σ_s, when the cylinder is in plastic state.

Substituting the Eqs. (2a) and (2b) into Eq. (5) obtained the shear stress Eq. (6) as follows:

$$\tau = \frac{(p_w - p_m) r_i^2 r_c^2}{(r_c^2 - r_i^2) r^2} \tag{6}$$

When the r was equal to r_i, the τ obtained the maximum on the inner wall of the internal cylinder, as follows:

$$\tau_{max} = \frac{(p_w - p_m) r_c^2}{(r_c^2 - r_i^2)} \tag{7}$$

Substituting the Eqs. (3a) and (3b) into Eq. (5) obtained the Eq. (8a) of the shear stress in plastic zone, as follows:

$$\tau^{rp} = 0.5\sigma_s - p_p(r_i/r)^2/[1-(1/K_i)^2] \tag{8a}$$

According to Eq (8a), the τ^{rp} obtained the maximum on the inner wall of the internal cylinder, when the r was equal to r_i.

Substituting the Eqs. (4a) and (4b) into Eq. (5) obtained the Eq. (8b) of the shear stress in plastic zone, as follows:

$$\tau^{re} = [(\sigma_s r_m^2 / 2r_c^2) - p_p /(K_i^2 - 1)](r_c^2 / r^2) \quad (8b)$$

According to Eq (8b), it can be known that the τ^{re} reached a maximum on the inside wall of the internal cylinder, when the r was equal to r_i.

According to Eqs. (8a)- (8b), we can obtain the following equation:

$$\tau^{rp} - \tau^{re} = 0.5\sigma_s - p_p (r_i / r)^2 /[1 - (1/K_i)^2]$$

$$-[(\sigma_s r_m^2 / 2r_c^2) - p_p /(K_i^2 - 1)](r_c^2 / r^2) = 0.5\sigma_s - \sigma_s r_m^2 / 2r_c^2 \quad (9)$$

Since, $r_i^2 < r_m^2 < r_c^2$, so Eq. (9) satisfied the following inequality:

$$\tau^{rp} - \tau^{re} = 0.5\sigma_s - \sigma_s r_m^2 / 2r_c^2 > 0 \quad (10)$$

That is, τ^{rp} is always greater than τ^{re}_{max} .

Substituting the r=r_i into Eq. (8a) obtained the equation as follows:

$$\tau^{rp}_{max} = 0.5\sigma_s - p_p /[1 - (1/K_i)^2] \quad (11)$$

In order to avoid reverse yielding, Ref. (Wang, Yuan, Hu, Wang, & He, 2007) gives the formula as follows:

$$p_{pmax} = \sigma_s (1 - 1/K_i^2) \quad (12)$$

Substituting the Eq. (12) into Eq. (11) obtained the equation as follows:

$$\tau^{rp}_{max} = 0.5\sigma_s - \sigma_s (1 - 1/K_i^2)/(1 - 1/K_i^2) = -0.5\sigma_s \quad (13)$$

Substituting the Eqs. (1a) and (1b) into Eq. (5) obtained the equation as follows:

$$\tau^{total} = (\sigma_\theta^{total} - \sigma_r^{total})/2$$

$$= (\sigma_\theta - \sigma_r)/2 + (\sigma_\theta^r - \sigma_r^r)/2 = \tau + \tau^r \quad (14)$$

where τ^{total} was the resultant shear stress.

From the inequality (10), it was known that τ^{rp} was always greater than τ^{re}, therefore the τ^{rp}_{max} was also always greater than the τ^{re}_{max} . Substituting the Eqs. (7) and (11) into Eq. (5) obtained the resultant maximum shear stress τ^{total}_{max} , as follows:

$$\tau^{total}_{max} = \tau_{max} + \tau^r_{max} = \frac{(p_w - p_m) r_c^2}{(r_c^2 - r_i^2)} - 0.5\sigma_s \quad (15)$$

When the inside wall of the internal cylinder just happen to yield, based on Eq. (5), τ^{total}_{max} is equal to $0.5\sigma_s$, that is,

$$\tau^{total}_{max} = \frac{(p_w - p_m) r_c^2}{(r_c^2 - r_i^2)} - 0.5\sigma_s = 0.5\sigma_s \quad (16)$$

According to Eq. (16), the elastic-limit pressure equation of the internal cylinder was expressed as below:

$$p_w = \sigma_s \left(1 - \left(\frac{1}{K_i}\right)^2\right) + p_m \quad (17a)$$

The external cylinder was only underwent the interlayer pressure p_m, meanwhile, it was also a conventional thick cylinder. According to Ref. (Wang, Yuan, Hu, Wang, & He, 2007), the plastic-limit pressure equation of the external cylinder was expressed as follows:

$$p_m = \frac{\sigma_s}{2}\left(1 - \left(\frac{1}{K_o}\right)^2\right) \quad (17b)$$

On the basis of Eqs (17a) and (17b), the plastic-limit workload p_w can be obtained, and it is obvious that the elastic-limit pressure p_w can approach 1.5 times yield strength σ_s of the materials, when K_o tends to infinity.

2.3 structrual optimum model

According to Eqs.(17a) and (17b), Eqs(18a) and (18b) were obtained respectively as follows:

$$K_i = [\sigma_s / (p_m + \sigma_s - p_w)]^{0.5} \quad (18a)$$

$$K_o = [\sigma_s / (\sigma_s - 2p_m)]^{0.5} \quad (18b)$$

$$K = K_i K_o \quad (18c)$$

where Eq.(18c) expressed the dimension relation between the radii, and K was total radius ratio: the outside radius r_o of the external cylinder to internal radius r_i of the internal cylinder.

Objective fuction (19a) can be derived by substituting the Eqs.(18a) and (18b) into Eq.(18c).

$$K = [\sigma_s / (p_m + \sigma_s - p_w)]^{0.5} \cdot [\sigma_s / (\sigma_s - 2p_m)]^{0.5} \quad (19a)$$

$$[\sigma_s / (p_m + \sigma_s - p_w)]^{0.5} > 1 \quad (19b)$$

$$[\sigma_s / (\sigma_s - 2p_m)]^{0.5} > 1 \quad (19c)$$

where Eqs.(19b) and (19c) were dimension constraint of radius ratios: K_i and K_o.

Eqs.(19a)-(19c) were the optimization model. Based on the above model, it can be obtained that the relation cureves between radius ratio and interlayer pressure under a specified working pressure.

3 Results and analysis

Suppose the material (tool steel H13) of cylinders in the ACCIP method were elastic-perfectly plastic, and its mechanical properties were as follows: the yield strength is 1542 MPa, the ultimate strength is 1884 MPa and the Possion's ratio is 0.3. If the elastic-limit workload p_w equaled 1650 MPa, and then the stresses of the ACCIP method under different K_i of 5.45, 2.83, 2.16, 1.81 and 1.59 were simulated.

The internal cylinder was autofrettaged before use, when it was autofrettaged, its residual stresses distribution of the internal autofrettaged cylinders was shown in Fig. 3 (a), (b) and (c). The end abscissas of the curves was the radius ratio r_c/r_i of the internal autofrettaged cylinders. It can be seen in Fig. 3 (a) that the maximum absolute value of the residual radial stress was less than 250 MPa, and when K_i=5.45, 2.83, 2.16, the internal autofrettaged cylinder was basically in full plastic state, therefore it can be seen in Fig. 3 (b) that there was obvious difference in the residual hoop stress between K_i=5.45, 2.83, 2.16, 1.81 and 1.59. Meanwhile, it can be seen in Fig. 3 (c) that all the residual shear stresses were lower than the shear yield of its materials, therefore it didn't happen to yield reversely. When the workload of the 1650 MPa was applied on the inner surface of the internal autofrettaged cylinder, the resultant shear stresses of whole apparatus was below than the shear yield strength line shown in Fig. 2.

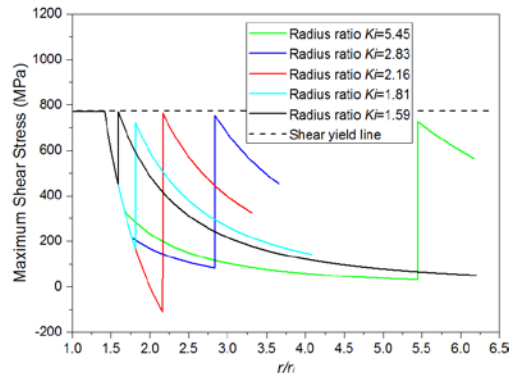
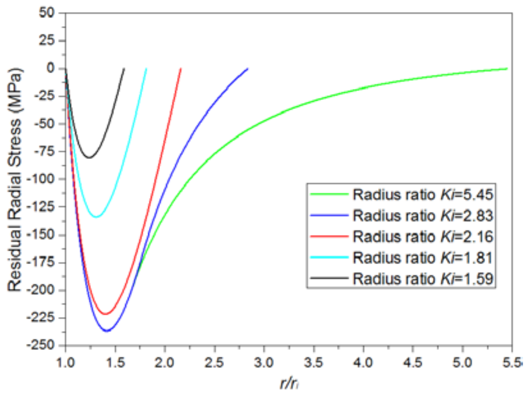
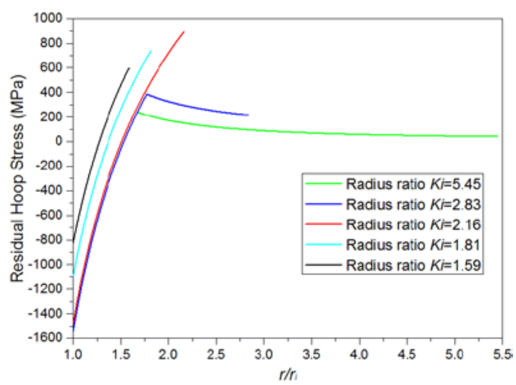


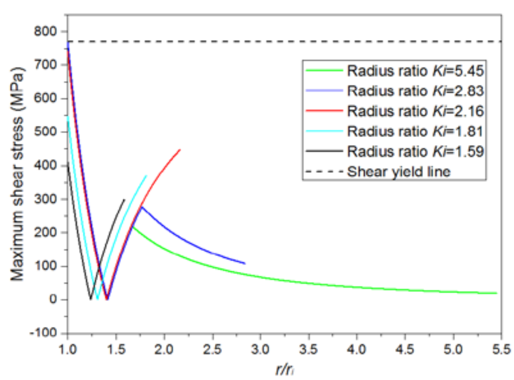
Fig. 2 Resultant shear stress of the cylinder when working pressure $p_w = 1650\text{MPa}$



(a) Residual radial stress



(b) Residual hoop stress



(c) Residual shear stress

Fig. 3 Residual stress of the internal autofrettaged cylinder

It can be seen from the above Fig. 2, when the workload p_w exceeded the yielding strength, the whole apparatus do not happen to yield; furthermore, outside radius r_o reached the minimum, when the radius ratio K_i was approximately equal to 2.16.

Interlayer pressure has effect on resultant shear stresses of the apparatus. Suppose the radius ratio K_i was approximately equal to 2.16, and the total radius ratio K reached the minimum 3.29. The interlayer pressures were 160, 300, 440, 580 and 720 MPa, respectively, and when the workload of 1650 MPa ($p_w = 1.07\sigma_s$) was applied on the inner surface of internal cylinder, the resultant shear stresses in the internal cylinder were shown in Fig. 4. When interlayer pressure p_m was 580 and 720 MPa, the corresponding resultant shear stresses were above than the shear yield strength limit and the external cylinder happened to yield; when interlayer pressure p_m was 160 and 300 MPa, the corresponding resultant shear stresses of the external cylinder were below than its shear yield strength, although the external cylinder didn't happen to yield, the internal cylinder did; and only when the interlayer pressure p_m was equal to 440 MPa, did neither the internal nor the external cylinder happen to yield. From the above analyses, the conclusion can be drawn that the interlayer pressure of the $p_m = 440$ MPa was the optimum interlayer pressure under the given working pressure of the $p_w = 1650\text{MPa}$, which avoid both the yield of the external cylinder and of the internal cylinder.

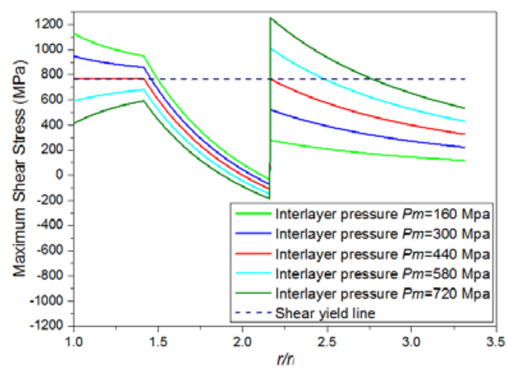


Fig. 4 Resultant Shear Stress vs interlayer pressure when working pressure $p_w = 1650\text{MPa}$

5 Conclusions

Many studies centered on shrinkage and autofrettage methods. Nevertheless, less attention has been payed to the ultrahigh pressure apparatus with interlayer pressure (ACCIP method). Therefore, the present work presented the general theoretical study of the ACCIP method.

In the present work, calculation of the stresses were on the basis of the ideal conditions, and the calculated results revealed the plastic-limit workload p_w can be as high as 1.5 times the yielding strength σ_s, which was much larger than those of shrink-fit vessel and autofrettaged cylinder. When the working pressure of the $p_w = 1.07\sigma_s$, the apparatus can remain within the elastic domain of the materials;

furthermore, the minimum radius ratio r_o/r_i was equal to 3.29. Analyses also showed that under the minimum radius ratio, the residual and resultant stresses were lower than the yielding strength respectively. Meanwhile, the studies also indicated that there was an optimum interlayer pressure when the total radius ratio takes the minimum. Overall, the ACCIP method can both increase elastic-limit workload of ultrahigh pressure apparatus and optimize its structure.

Acknowledgments: This work was supported by the Natural Science Foundation of Jiangsu Province of China (Grant No. BK20200999), meanwhile I am also grateful to Prof. Renshu Yuan for his help.

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