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Uncertainty-based Multidisciplinary Design Optimization using An Approximated Second-Order Reliability Analysis Strategy

Zhiyuan LYU[1], Hongtao WANG[1], Hengfei YANG[1], Jiapeng WANG[1], Ketema Mikiyas Solomon[1], Debiao MENG[1,2,3*]

1. School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
2. Institute of Electronic and Information Engineering of UESTC in Guangdong, Dongguan 523808, China
3. Yangzhou Yangjie Electronic Technology Co., Ltd., Jiangsu Yangzhou 225008, China

***Corresponding Author:** Debiao Meng, E-mail address: dbmeng@uestc.edu.cn*

Abstract:
In uncertainty-based multidisciplinary design optimization (UBMDO), all reliability limitation factors are maintained due to minimize the cost target function. There are many reliability evaluation methods for reliability limitation factors. The second-order reliability method (SORM) is a powerful most possible point (MPP)-based method. It can provide an accurate estimation of the failure probability of a highly nonlinear limit state function despite its large curvature. But the Hessian calculation is necessary in SORM, which results in a heavy computational cost. Recently, an efficient approximated second-order reliability method (ASORM) is proposed. The ASORM uses a quasi-Newton method to close to Hessian without the direct calculation of Hessian. To further improve the UBMDO efficiency, we also introduce the performance measure approach (PMA) and the sequential optimization and reliability assessment (SORA) strategy. To solve the optimization design problem of a turbine blade, the formula of MDO with ASORM under the SORA framework (MDO-ASORM-SORA) is proposed.
Keywords: *uncertainty; reliability analysis; optimization design; turbine blade*

Introduction

The Multidisciplinary design optimization (MDO) is a methodology for the design problems of complicated and coupled engineering systems, which has received extensive attention from industry and academia [1-11]. The application of MDO research results has expanded from the initial hypersonic aircraft, large passenger aircraft, shuttle spacecraft, and other aerospace fields to vehicles and ships, electronics, energy, and civil and construction and other engineering fields have produced significant technical and economic benefits [12-16]. To effectively consider the influence of these uncertain factors in the process of design optimization, uncertainty-based multidisciplinary design optimization (UBMDO) has become one of the research hot spots of modern engineering system design [17-26]. So far, the UBMDO method that considers random uncertainty has become more mature after combining reliability analysis methods such as classical probability theory. Due to the adoption of the sequence optimization and reliability evaluation sequential optimization and reliability assessment (SORA) strategy, the reliability analysis process and the design optimization process are decoupled [27-29]. The entire

UBMDO process is decomposed into a series of alternate deterministic MDO and reliability analysis processes, and the computational efficiency is further improved [30-31].

However, the Hessian calculation is necessary in second-order reliability method (SORM), which results in a heavy computational cost. To further improve the efficiency and robustness of UBMDO, based on UBMDO, the approximated second-order reliability method (ASORM) method based on performance measure approach (PMA) under SORA strategy is proposed.

1 Traditional reliability calculation method

1.1 First order reliability method (FORM)

For the limit state equation $G(x)=0$, x is

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represented as a vector form of a random variable in the original space, which is transformed into an independent standard normal space (U space), and the limit state equation will be rewritten as $g(u)=0$. According to the physical meaning of the reliability index, the problem of calculating the reliability index is transformed into the problem of solving the minimum distance from the origin of the coordinate to the limit state surface in U space. The specific constrained optimization problem is as follows:

When $g(u)=0$, compute the $u^*=\min_n \|u\|$, So the first-order reliability index β^{FORM} and the instability probability P_f^{FORM} are computed by Eq. (1):

$$\begin{aligned} \beta^{\mathrm{FORM}} &= \|u^*\| \\ P_f^{\mathrm{FORM}} &\approx \Phi\left(-\beta^{\mathrm{FORM}}\right) \end{aligned} \tag{1}$$

where $\|\bullet\|$ is the modulus of the vector, Φ is the standard normal distribution function.

1.2 The review of SORM

The SORM method uses Taylor series to expand the function $g(u)$ at point u^*, and keeps the quadratic term. The second-order approximate expression of $g(u)$ can be obtained as follow:

$$g(u)=\beta^{\mathrm{FORM}}-\alpha^T u+\frac{1}{2}\left(u-u^*\right)^T H\left(u-u^*\right) \tag{2}$$

where $\alpha=-\frac{\nabla g\left(u^*\right)}{\left|\nabla g\left(u^*\right)\right|}$ is the unit direction vector at the verification point in U space, $H=\frac{\nabla^2 g\left(u^*\right)}{\left|\nabla g\left(u^*\right)\right|}$ is the n-dimensional Hessian matrix, n is the number of random variables, $\nabla g(u)$ is the gradient vector of the verification point u^*, which can use the front difference method to calculate:

$$\nabla g(u)_i=\frac{g\left(u_i+\Delta h\right)-g\left(u_i\right)}{\Delta h} \tag{3}$$

where Δh is the step size, i is the i th component of the vector.

On the basis of the obtained verification point u^* and the Hessian matrix H, the random variable is transferred from U space to V space to ensure that the n th axis of V space coincides with the verification point vector u^*, which can be solved by the Gram-Schmidt method. The matrix form is V=PU. Operate the rotation matrices P and H, select the first n-1 order to form a sub-matrix, and perform diagonal processing on the sub-matrix to obtain the n-1 order diagonal matrix H_{rot} as follows:

$$H_{rot}=P\frac{H}{\left\|\nabla g\left(u^*\right)\right\|}P^T \tag{4}$$

The diagonal element of the matrix H_{rot} is the principal curvature κ_i, which is $\kappa_i=\left[H_{rot}\right]_{ii}(i=1,2,\ldots,n-1)$. Geometrically, κ_i represents the principal curvature at the check point. After obtaining the check point and principal curvature, different algorithms can be used to calculate the second-order reliability.

2 The approximated second-order reliability method

In UBMDO, the evaluation of the reliability constraints can be defined using a multidimensional integral. However, when the limit state functions are nonlinear, the multidimensional integral cannot be calculated analytically. Therefore, we approximate the limit state function by the Taylor series of second-order at the MPP in U-space.

To eliminate the mistake caused by quadratic function parabolic approximation and obtain a better accuracy, this study uses the SORM with the generalized chi-square distribution.

In practical engineering problems, the calculation of Hessian analysis cannot be performed. To solve previous problem, utilizing the quasi-Newton approach to approximate the Hessian. This paper also introduces the ASORM.

When being close to the N×N Hessian matrix, taking a symmetric matrix, there are N(N+1)/2 degrees of freedom, while the secant line term exerts only N constraints. the unique Hessian updates required additional constraints. The symmetric rank-one (SR1) updates and creates the unique symmetric matrix with a rank-one amendment meeting the secant qualification.

Finding a FORM calculation method with good convergence is the basis for calculating SORM indicators. In this paper, the HLRF-BFGS algorithm [32] is used to determine the check point to obtain the first-order reliability index; then the SR1 algorithm is used to approximate the Hessian matrix to obtain the second-order reliability index with excellent accuracy.

Based on the FORM calculation process of the HLRF-BFGS algorithm, the search calculation direction d_k can be obtained by the Eq. (5):

$$d_k=\frac{\left[\nabla g\left(u_{k-1}\right)^T B_{k-1}^{\mathrm{BFGS}}u_{k-1}-g\left(u_{k-1}\right)\right]}{\nabla g\left(u_{k-1}\right)^T B_{k-1}^{\mathrm{BFGS}}\nabla g\left(u_{k-1}\right)} B_{k-1}^{\mathrm{BFGS}}\nabla g\left(u_{k-1}\right)-B_{k-1}^{\mathrm{BFGS}}u_{k-1} \tag{5}$$

$$B_k^{\mathrm{BFGS}} = B_{k-1}^{\mathrm{BFGS}} + \left(1 + \frac{q_k^T B_{k-1}^{\mathrm{BFGS}} q_k}{d_k^T q_k}\right)\frac{d_k d_k^T}{d_k^T q_k} - \frac{d_k q_k^T B_{k-1}^{\mathrm{BFGS}} + B_{k-1}^{\mathrm{BFGS}} q_k d_k^T}{d_k^T q_k} \tag{6}$$

where k is the number of iterations; B^{BFGS} is the inverse matrix of the Hessian matrix calculated by the BFGS algorithm, that is $B^{\mathrm{BFGS}} = \left(H^{\mathrm{BFGS}}\right)^{-1}$, which can be obtained by the recursive Eq. (6). q_k is expressed as Eq. (7):

$$q_k = d_k + \left[\nabla g\left(u_k\right) - \nabla g\left(u_{k-1}\right)\right]\xi_k \tag{7}$$

where $\xi_k = \dfrac{g\left(u_{k-1}\right) - \nabla g\left(u_{k-1}\right)^T B_{k-1}^{\mathrm{BFGS}} u_{k-1}}{\nabla g\left(u_{k-1}\right)^T B_{k-1}^{\mathrm{BFGS}} \nabla g\left(u_{k-1}\right)}$.

On this basis, the check point in the new iteration can be obtained by the Eq. (8):

$$u_k = u_{k-1} + d_k \tag{8}$$

Repeat the above iteration process, the iteration stops when the following conditions are met: $1 - \dfrac{\left|\nabla g\left(u_k\right)^T u_k\right|}{\left\|\nabla g\left(u_k\right)\right\| \cdot \left\|u_k\right\|} < \varepsilon$ and $\left|g\left(u_k\right)\right| < \varepsilon$.

At the same time, the SR1 algorithm can provide a more accurate approximation of the Hessian matrix than other methods. Therefore, SR1 algorithm is used to approximate the Hessian matrix in each iteration in this study. Its expression is as follow:

$$H_k^{\mathrm{SR1}} = H_{k-1}^{\mathrm{SR1}} + \frac{\left(y_k - H_{k-1}^{\mathrm{SR1}} d_k\right)\left(y_k - H_{k-1}^{\mathrm{SR1}} d_k\right)^T}{\left(y_k - H_{k-1}^{\mathrm{SR1}} d_k\right)^T d_k} \tag{9}$$

where $y_k = \nabla g\left(u_k\right) - \nabla g\left(u_{k-1}\right)$.

Compared with SORM, ASORM adopts the approximated Hessian rather than direct calculation of Hessian. Thereby it requires computations only used in FORM to realize much efficient and precise reliability analysis. Compared with FORM, in the most possible point (MPP) search, ASORM makes full use of the information collected, Thus the reliability assessment can be more accurate.

3 Review of PMA and SORA

3.1 The PMA

In UBMDO, adopting PMA is more effective than direct evaluation of actual probability. If some non-active reliability restrictions are directly evaluated to get their real probabilities, they will govern the entire calculation process, leading to low computational efficiency.

The basic formula of UBMDO is to minimize the objective function under the restriction of probability, which can be expressed as Eq. (10).

$$\begin{aligned}
&\min_{(d_s, d_i, x_s, x_i)} \ f\left(d_s, d_i, x_s, x_i\right),\\
&s.t.\quad J_1\left(d_s, d_i, x_s, x_i\right) \le \varepsilon,\\
&\qquad J_2\left(d_s, d_i, x_s, x_i\right) \le \varepsilon,\\
&\qquad \Pr\left\{g_i\left(d_s, d_i, x_s, x_i\right) \le 0\right\} \ge R\\
&\qquad d_s^L \le d_s \le d_s^U,\\
&\qquad d_i^L \le d_i \le d_i^U,\\
&\qquad x_s^L \le x_s \le x_s^U,\\
&\qquad x_i^L \le x_i \le x_i^U\\
&\qquad i = 1, 2
\end{aligned} \tag{10}$$

where the $f(\bullet)$ indicates the objective function of UBMDO problem. di represents the local design variables for the i th discipline. ds are shared design variables in all disciplines. xi express the local input variables for discipline i, xs denotes the vector of sharing variables which are input variables of every discipline. $\Pr\left(g^{(i)} > 0\right) \le \alpha_t$ are probability restrictions to discipline i. $\Pr\left(g > 0\right)$ refers to the probability of fail with the pattern of $g > 0$. J1 and J2 are the flabby limitation requirements of subsystem 1 and subsystem 2, severally. ε represents an extremely small positive number that can be dynamically changeable. gi and Ri signify the probability limitation requirements and allowed reliability separately.

To every probability restriction, the PMA-based UBMDO can be depicted as Eq. (11).

$$\begin{aligned}
&\min_{\left(u_s^{(i)}, u_i^{(i)}\right)} \ g^{(i)}\left(d_s, d_i, u_s^{(i)}, u_i^{(i)}\right)\\
&s.t.\quad \left\|\left(u_s^{(i)}, u^{(i)}\right)\right\| = \beta_t\\
&\qquad i = 1, 2, \mathrm{L}\ , n
\end{aligned} \tag{11}$$

where $u_s^{(i)}$ and $u_i^{(i)}$ are the standard normal stochastic variables for U-space, they separately denote stochastic variables of x_s and x_i of discipline i in X-space. β_t is the required reliability. $u^{(i)}$ consists of all standard normal stochastic variables in all disciplines.

3.2 The SORA strategy

In UBMDO, The SORA adopts series of cycles for reliability analysis and decoupled deterministic MDO. In every cycle, reliability analysis and MDO are mutually decoupled. Reliability analysis is performed after MDO, and the process of SORA is demonstrated. The ASORM is used to work out the reliability estimation problems in a cycle in this study. Use PMA to construct a new deterministic MDO problem for the next cycle within the framework of SORA.

4 The turbine blade design optimization

In this paper, the UBMDO which only considers interval variables is improved, and both interval variables and random variables are considered in the optimization design process. Then, taking the optimal design of the planetary gearbox for a megawatt wind turbine as the research object, the uncertainty factors in the optimal design of the wind turbine planetary gearbox are analyzed. On this basis, the proposed method is used to optimize the design of the planetary gearbox. The final comparative analysis shows the result of planetary gearbox optimization, and the optimization design scheme of this paper is feasible.

Turbine blades are important components which make up the turbine section of a gas turbine [33-34]. The blades extract energy from the high temperature, high pressure gas produced by the combustor, thus the turbine blade design optimization is a typical MDO problem including heat transfer, aerodynamics and structure. In this study, a turbine blade design optimization problem is solved using the proposed UBMDO method. The objective is maximizing the aerodynamic efficiency η. There are nine design variables and three constraints, which are shown in Figure 1 and Tab. 1.

Here, we assume that all design variables are random variables which are normally distributed. We use FORM, SORM and ASORM to solve this MDO problem, respectively. In Tab. 2, we can see that all reliability estimation methods can obtain reasonable solutions. The UBMDO methods using ASORM and using FORM require almost the same computation time t. However, the aerodynamic efficiency from UBMDO using ASORM are more conservative than that from UBMDO using FORM. Compared with the aerodynamic efficiency η=0.9428 from UBMDO using SORM, the aerodynamic efficiency from UBMDO using ASORM is 0.9468. It means that two UBMDO methods can enjoy higher accurate reliability estimations.

Table 1 Design variables and constraints of turbine blade design optimization

			Description	Initial value	Lower bound	Upper bound
Design variables	Top section	$r_1/r_2/r_3$	Installation angle	41.0/61.0/74.0	38.0/58.0/71.0	44.0/64.0/77.0
	Middle section	$\varphi_1/\varphi_2/\varphi_3$	Incidence angle	10.0/2.0/-6.0	7.0/0.5/-9.0	13.0/5.0/-3.0
	Root section	$\xi_1/\xi_2/\xi_3$	Deviation angle	6.0/4.0/2.0	3.0/1.0/0.5	9.0/7.0/5.0
Constraints	Maximum temperature/K			983.22	-	1000
	Equivalent stress/MPa			603.09	-	120
	Maximum deformation/mm			0.5012	-	0.6

Table 2 Design variables and constraints of turbine blade design optimization

	r_1	φ_1	ξ_1	r_2	φ_2	ξ_2	r_3	φ_3	ξ_3	η	t
ASORM	38.17	11.09	6.56	57.08	4.66	6.15	71.97	-4.97	1.06	0.9468	72hr/43min
SORM	38.42	11.23	6.37	56.35	4.71	6.44	71.35	-4.58	1.10	0.9428	129hr/39min
FORM	36.34	10.34	6.53	55.30	4.89	6.92	73.01	-4.55	1.28	0.9613	69hr/39min

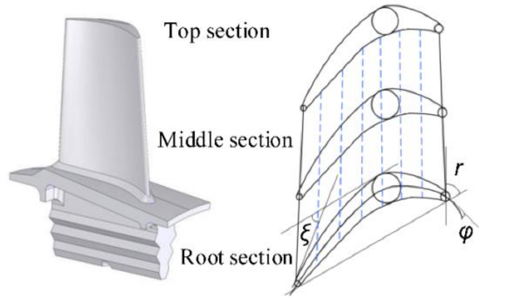

Figure 1 The structure sketch of a turbine blade

5 Conclusion

In UBMDO, the computational cost of the objective function is minimized while preserving all reliability constraints. There are many methods of reliability assessment. SORM is an MPP-based method. It can accurately estimate the failure probability of highly nonlinear limit state functions. However, Hessian calculation is required in SORM, and the calculation cost is very high. Recently, an efficient ASORM has been proposed. ASORM uses the quasi-Newton method to approximate the Hessian without directly computing the Hessian. To further improve UBMDO efficiency, we also introduce PMA and SORA strategies. To solve the optimal design problem of turbine blades, MDO-ASORM-SORA is proposed. It is demonstrated that the proposed method has more accurate reliability estimates.

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