

# Reliability analysis of small failure probability based on subset simulation method

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## Abstract:

In the engineering, to ensure the quality and safety, it is necessary to carry out reliability analysis on it. When conducting reliability analysis in engineering, a large number of small failure probability problems will be encountered. For such problems, the traditional Monte Carlo method needs a lot of samples, and the calculation efficiency is extremely low, while the subset simulation method can efficiently estimate the reliability index of the small failure probability problem with little samples. Therefore, this paper takes the application of the subset simulation method in the reliability analysis of the small failure probability structure as the object, constructs the reliability analysis method of the single failure mode of the system, and applies the method to a mathematical example and a single-story gate. Through the rigid frame example, it can be seen that this method is beneficial to improve the calculation efficiency and accuracy.

**Keywords:** subset simulation; small failure probability; failure mode; reliability analysis

## 1 Introduction

Reliability engineering <sup>[1]</sup> is a comprehensive engineering discipline that includes various engineering technologies, including statistics and analysis of product failures and their probability of occurrence, reliability design <sup>[2-3]</sup>, reliability prediction <sup>[4-6]</sup>, reliability test <sup>[7]</sup>, reliability evaluation <sup>[8-9]</sup>, reliability inspection <sup>[10-11]</sup>, reliability control <sup>[12]</sup>, reliability maintenance <sup>[13-14]</sup> and failure analysis <sup>[15-17]</sup>. Its essence is to fight against failure or malfunction, throughout the whole life cycle of the product. For a product, the high reliability of the system is particularly important. In the engineering system, because all parts work with each other, if some parts fail, it will usually cause serious accidents <sup>[18-21]</sup>. Therefore, when designing such a system, it needs to have a low failure probability to guarantee that the engineering system has higher safety. For example, in the field of aviation and aerospace, the British Aviation Commission stipulates that the failure rate of aircraft must be lower than  $10^{-5}$  <sup>[22-23]</sup>. In the field of automobile industry, there is a key index in the process of automobile design, that is, the safety of automobile structure. According to the design standard of automobile fatigue reliability, the failure probability of key structure of car body should be less than 0.01% <sup>[24-25]</sup>. Therefore, to achieve high-precision evaluation, it is necessary to study the analysis method of Small Failure

Probability (SFP) problem <sup>[26-27]</sup>.

SFP means that the probability of part failure is very small. So it is almost impossible to occur in one test.

However, it will inevitably occur in repeated tests. The reliability analysis method of SFP problem is mainly sampling method <sup>[28-29]</sup>. The Monte Carlo Simulation (MCS) method has strong versatility, the calculation accuracy increases with the increase of the sampling point capacity. Reliability analysis results with the required accuracy can be obtained theoretically. However, when the dimension of the random variable is too large, the vast majority of the sample points extracted by the MCS method will fall into the safety domain of the design space, which makes little contribution to the reliability analysis of failure events. In addition, the computational cost of obtaining sample points for high-dimensional and highly nonlinear systems is high, which leads to the low efficiency of the MCS method. If the utilization rate of sample points can be improved, especially in the analysis of multi-disciplinary complex systems, the calculation cost will be greatly reduced. For SFP problems in large and complex engineering systems, the subset simulation method (SS) can maintain considerable accuracy and efficiency in calculations <sup>[30-32]</sup>. SS is an efficient and accurate method for calculating SFP. Its core is to use adaptive method to decompose the whole large probability space into a series of nested subspaces. These

subspaces are regarded as intermediate failure events, which correspond to a series of conditional failure probabilities. According to Bayesian formula [33-35], these conditional failure probabilities are multiplied in turn, then the tiny failure probability of the required solution is obtained. This method improves the efficiency and accuracy of the SFP problem [36-38].

Abdollahi and Moghaddam et al [39], introduced the Subset Control Variable (SCV) technology, which is a new method to reformulate the traditional SS and provides statistical features. Au and Beck [40] expressed the failure probability as the product of the larger conditional failure probability, and proposed a new simulation method SS to calculate the SFP encountered in the reliability analysis of engineering systems. Xiao and Zhang et al. [41] proposed an effective Kriging-based Subset Simulation (KSS) method for mixed reliability analysis under random and interval variables using SFP. Qian and Li et al. [42] proposed a time-varying system reliability analysis method, combining multi-response Gaussian process (MRGP) and SS to solve the SFP issues.

The structure of this article can be summarized as follows: The second section reviews the Markov chain Monte Carlo method (MCMC). The third section introduces the principle and calculation process of the SS. In the fourth section, two examples are used to verify the advantages of the SS in solving the SFP. Finally, the content of the full text is summarized in the fifth section.

## 2 Review of Markov Chain Monte Carlo

MCS [43] is a calculation method based on random numbers. Assuming that the domain of the random variable  $x$  is  $X$ , and its probability density function (PDF) is  $f(x)$ . the objective of MCS is to solve the mathematical expectation  $E_{f(x)}[g(x)]$ .  $g(x)$  is the function defined on  $X$ . MCS independently samples  $n$  samples  $x_1, x_2, \dots, x_n$  according to  $g(x)$ , the approximate expectation is:

$$E_{f(x)}[g(x)] \approx \frac{1}{n} \sum_{i=1}^n g(x_i) \quad (1)$$

Assuming that the integral of solution  $z(x)$  on  $X$  is required:

$$\int_X z(x) dx \quad (2)$$

It is necessary to decompose  $z(x)$  into the  $g(x)$  and  $f(x)$ , and then transform the problem into solving the mathematical expectation  $E_{f(x)}[g(x)]$  of the  $g(x)$  about  $f(x)$ :

$$\int_X z(x) dx = \int_X \frac{z(x)}{f(x)} f(x) dx = \int_X g(x) f(x) dx = E_{f(x)}[g(x)] \quad (3)$$

Then

$$\int_X z(x) dx = E_{f(x)}[g(x)] \approx \frac{1}{n} \sum_{i=1}^n g(x_i) \quad (4)$$

In many cases, the performance function  $z(x)$  of the system structure is complex and can't be directly used to calculate the failure probability, and the

above-mentioned MCS can better solve this problem. However, sometimes the decomposed PDF  $g(x)$  is equally complex, and MCS is no longer applicable.

MCMC [44] can efficiently generate a series of samples that obey the complex distribution  $f(x)$ . Suppose the state space of Markov chain (MC)  $(X = X_0, X_1, \dots, X_t, \dots)$  is  $S$ . Moreover, the transition probability matrix is  $P = (P_{ij})$ , where  $P_{ij}$  denotes the probability that the  $j$ th state will be transferred to the  $i$ th state. If there is a distribution  $\pi = (\pi_1, \pi_2, \dots)T$  on the state space  $S$ , such that  $\pi = P\pi$ , then the stationary distribution of MC is  $\pi$ .

If an original distribution is the stationary distribution of the MC, after any transfer operation, its result is still the stationary distribution. If the MC is aperiodic, irreducible, and returns normally, its stationary distribution  $\pi = (\pi_1, \pi_2, \dots)T$  is unique, and its limit distribution is the stationary distribution. According to the ergodic theorem, no matter what the initial distribution  $\pi'$  is, after  $n$  transitions, that is  $P^n \pi'$ , it will eventually converge to its stationary distribution  $\pi$ , that is:

$$\lim_{t \rightarrow +\infty} P(X_t = i | X_0 = j) = \pi_i \quad i \in N^+; j \in N^+ \quad (5)$$

For any time  $t$  and any state  $i, j \in S$ , the state distribution satisfies the following equation:

$$P(X_t = i | X_{t-1} = j) \pi_j = P(X_{t-1} = j | X_t = i) \pi_i, i, j \in N^+ \quad (6)$$

or abbreviated as:

$$p_{ji} \pi_j = p_{ij} \pi_i, i, j \in N^+ \quad (7)$$

This is the meticulous equilibrium equation.

The core idea of MCMC method is divided into the following 3 steps. Firstly, define an MC  $\pi = (\pi_1, \pi_2, \dots)T$  in the state space  $S$  of the random variable  $X$  to make its stable distribution as the sampling target distribution  $f(x)$ . Secondly, randomly walk on this MC to get a sample at any time. Thirdly, find the mathematical expectation of the function according to the ergodic theorem. The meaning of the ergodic theorem is as follows: if the time tends to infinity and the sample distribution closes a stationary distribution, then the function mean of the sample is close to the mathematical expectation of the function.

In this paper, Metropolis-Hastings algorithm [45], is adopted to define MC and transition kernel. Suppose a probability distribution that needs to be sampled is  $f(x)$

$$p(x, x') = t(x, x') \beta(x, x') \quad (8)$$

where  $x'$  represents the candidate state;  $\beta(x, x')$  is called the accepted distribution;  $t(x, x')$  represents the transfer core of another MC; and it is called the suggested distribution.

$$\beta(x, x') = \min \left\{ 1, \frac{p(x') t(x', x)}{t(x, x') p(x)} \right\} \quad (9)$$

Since

$$\begin{aligned} p(x) t(x, x') &= p(x) t(x, x') \min \left\{ 1, \frac{p(x') t(x', x)}{t(x, x') p(x)} \right\} \\ &= \min \{ t(x, x') p(x), t(x', x) p(x') \} \\ &= t(x', x) p(x') \min \left\{ 1, \frac{t(x, x') p(x)}{t(x', x) p(x')} \right\} \\ &= p(x', x) p(x') \end{aligned} \quad (10)$$

satisfies the meticulous equilibrium equation. The transition kernel  $p(x',x)$  satisfies the ergodic theorem and the final stationary distribution is  $p(x)$ , that is, the samples generated in this way conform to the  $p(x)$  distribution.

It is suggested that  $t(x,x')=t(x',x)$ , then

$$\beta(x,x') = \min\left\{\frac{p(x')}{p(x)}, 1\right\} \quad (11)$$

Particularly,  $t(x,x')=t(|x-x'|)$  is called the random walk Metropolis algorithm, such as the normal distribution transition kernel:

$$t(x,x') \propto \exp\left(-\frac{(x'-x)^2}{2}\right) \quad (12)$$

Its characteristic is that when  $x'$  is closer to the mean value  $x$ , the acceptance probability is higher.

The steps of Metropolis-Hastings algorithm are:

I. According to the objective distribution function  $f(x)$  to be sampled, select a suggested distribution  $t(x,x')$  and randomly select an initial value  $x = x_0$ ;

II. According to the recommended distribution  $t(x,x')$ , a candidate state  $x'$  is randomly selected and the acceptance probability is calculated :

$$\beta(x,x') = \min\left\{1, \frac{p(x')t(x',x)}{t(x,x')p(x)}\right\} \quad (13)$$

III. A number  $m$  is randomly selected from interval  $(0, 1)$  according to uniform distribution. If  $m \leq \beta(x,x')$ , then accept the candidate state  $x'$ , otherwise, refuse to transfer.

In the SS of this paper, the candidate state  $x'$  newly generated by MCMC must meet the conditions  $x' \in F_i$ , to ensure that the newly generated state is within the failure domain  $F_i$  area.

### 3 Subset simulation method

SS is a random simulation process for estimating SFP. Specifically, consider an engineering system constrained by random input parameters. The failure area  $E$  is defined as the sub-area of the response function  $G(x)$  less than a certain threshold  $b$  in  $x$  space, that is:

$$E = \{x: G(x) < b\} \quad (14)$$

where  $x$  represents the random input vector of all uncertain parameters. In the SS,  $G(x)$  can be a nonlinear implicit function of  $x$ . In many cases, the target failure probability  $P_E$  related to the target failure event  $E$  may be small, so it is necessary to carry out much simulation to estimate the target failure probability to obtain the required accuracy, but it also reduces the computational efficiency. The SS method transforms an SFP into a product of a larger conditional probability sequence. This transformation method is to divide the input parameter space into subsets of fault domains. Therefore, it is necessary to define a series of intermediate failure events in the same way as the target failure event

$$E_j = \{x: G(x) < b_j\}, j = 1, \dots, m \quad (15)$$

where  $m$  represents the total number of intermediate

events, and  $b_j$  represents a set of thresholds of the system response function. Suppose  $E_1, E_2, \dots, E_m$  is a nested event sequence, that is,  $E_1 \supset E_2 \supset \dots \supset E_m = E$ , but the value of  $b_j$  cannot be predetermined. However, setting the conditional probability  $P(E_j|E_{j-1})$  equal to a specified value. To ensure the nesting of  $E_j$ , set the threshold value as  $b_1 > b_2 > \dots > b_m = 0$ . Because all intermediate events are nested, then

$$P_E = P(E_1) \prod_{j=2}^m P(E_j|E_{j-1}) \quad (16)$$

In the reliability analysis, the SS starts from the first step, and the probability  $P_1$  related to the first intermediate event  $E_1$  is calculated as follows:

$$P_1 = P(E_1) \approx \frac{1}{N} \sum_{i=1}^N I_{E_1}(G(x_i)) \quad (17)$$

where  $N$  denotes the total number of samples of the first intermediate event, that is, the first simulation layer;  $\{x_i\}$  represents a random input vector sequence of all uncertain parameters in the system generated according to known PDF, which is an independent and identically distributed sample.  $I_{E_1}(\cdot)$  is an index function:

$$I_{E_1}(\cdot) = \begin{cases} 0 & \text{if}(x_i) \geq b_1 \\ 1 & \text{if}(x_i) < b_1 \end{cases} \quad (18)$$

where the first intermediate event and its threshold are unknown, but if the conditional probability of each layer is set to a fixed value  $p_1$ , the values of threshold  $b_1$  and first intermediate event  $E_1$  can be determined according to Eq. (17). After generating  $\{x_i\}$ , all  $N$  system response functions  $\{G(x_i)\}$  are calculated and sorted in ascending order so that  $G(x_1) \leq G(x_2) \leq \dots \leq G(x_N)$ . Let  $b_1$  be the sample quantile of the system response function in the first layer simulation, that is  $b_1 = G(x_{[p_1 N]})$ , then samples  $x_1, x_2, \dots, x_{[p_1 N]}$  all belong to the first intermediate event  $E_1$ .

In the subsequent intermediate event  $E_j$ , the sample source is the previous intermediate event  $E_{j-1}$ . Considering that there are already  $[N \times p_{j-1}]$  samples that belong to  $E_{j-1}$ , you can use the sampling method based on MCMC to get the required conditional samples  $\{x_i\}$ . Then use  $P(E_j|E_{j-1})$  to perform probability estimation on the simulation layer:

$$P_j = P(E_j|E_{j-1}) \approx \frac{1}{N} \sum_{i=1}^N I_{E_j}(G(x_i)) \quad (19)$$

where the sample  $x_i \in f(x|E_{j-1}), i = 1, \dots, N$  is produced by MCMC. Generate  $N - [N \times p_{j-1}]$  conditional samples in the intermediate event  $E_{j-1}$  and combine them with the previously selected  $[N \times p_{j-1}]$  samples. All  $N$  system response functions  $\{G(x_i)\}$  can be calculated, and sorted in ascending order. Let  $b_j$  be the sample quantile of  $N$  system response values in event  $E_j$ , namely  $b_j = G(x_{[p_j N]})$ . In this way,  $x_1, x_2, \dots, x_{[p_j N]}$  all belong to intermediate event  $E_j$ .

Iterate the above steps repeatedly. When the sample quantile of  $N$  system response values in space  $E_{j-1}$  below  $b$ , that is  $b_j = G(x_{[p_j N]}) < b$ , stop the iteration. At this point, the SS algorithm has reached the target failure

domain  $E_m$ , that is,  $j = m$  and  $b_m = b$ . The final conditional probability estimates of  $P(E_m|E_{m-1})$  is:

$$P(E_m|E_{m-1}) \approx P_m = \frac{1}{N} \sum_{i=1}^N I_{E_m}(G(x_i)) \quad (20)$$

Combining Eq. (16), (17), (18) and (19)

$$P_E = \prod_{j=1}^m P_j \quad (21)$$

In the practical application, the value of  $P_j$ ,  $j = 1, \dots, m$  are usually  $P_j = p_0 \in (0.1 \sim 0.3)$ .

Therefore, the steps of applying SS in MATLAB can be briefly summarized as follows: I. Define the algorithm parameters; II. Using direct MCS to generate the first intermediate failure event; III. Using MCMC method to generate the remaining intermediate failure events; IV. Calculate the failure probability of the target event by multiplying the conditional probabilities of all intermediate failure events.

The algorithm flow is shown in Figure 1:

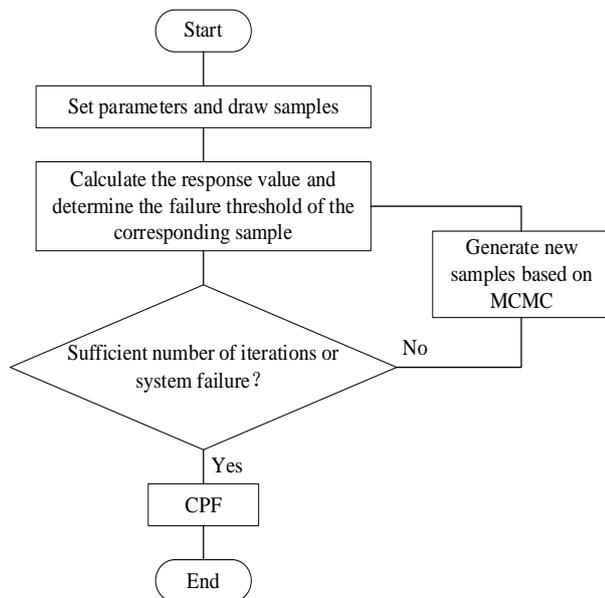


Figure 1 The flow chart of SS algorithm

## 4 Examples

### 4.1 The example 1

The functional function of the mathematical example has been given:

$$G(x) = 0.0185361 - \frac{73.8221x_1}{x_2^3} \quad (22)$$

where  $x_1 \sim N(1100, 201.5)$ ,  $x_2 \sim N(253, 38.1)$ , the failure mode is that the value of the function is below 0.

Use SS to calculate its reliability. In the setting of basic parameters, the total number  $N$  of samples is 2000, and the conditional probability of each simulation layer is set to  $p_0=0.25$ . By running the MATLAB program based

on SS, the consequences are shown in Table 1:

Table 1 shows the whole process of calculating the failure probability of the system response function by MATLAB. The program has implemented three random simulation layers, including one MCS layer ( $j=1$ ) and two MCMC simulation layers ( $j=2, 3$ ). The basis for stopping the cycle of this program is to judge the quantile of the current analog layer sample  $g(x_{[P_j N]})$  is less than zero. Of course, the third simulation layer meets this criterion and reaches the failure zone of the system failure mode, so the program exits the loop. Then, it is found that the number of all samples in the simulation layer whose response value is less than zero is 540, so the failure probability of the response function is  $9.2 \times 10^{-3}$ , and a total of  $2000+1500+1500=5000$  random samples are needed.

Table 1 The execution process of SS

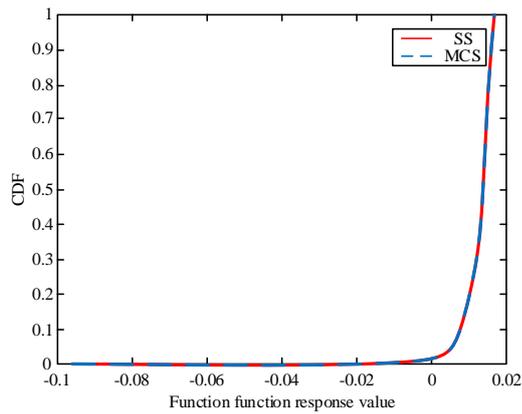
Simulation layer j	Seed number $N_j$	Number of samples generated in layer j $N-N_j$	Condition $P(U(j+1) U(j))$	The failure mode probability obtained by the J layer
0	0	2000		
1	500	1500	0.25	
2	500	1500	0.25	
3	540		0.27	$P(E1)=9.0 \times 10^{-3}$

For the rigor of the experiment, this paper also uses MCS to verify the accuracy of the result. After many attempts, it is found that the failure probability of the response function converges only when the total number of selected samples is above 105. The number of samples below this order of magnitude will affect the accuracy of the results, while the number above this order of magnitude will greatly affect the calculation efficiency. Therefore, we choose to randomly generate 200,000 samples in the whole sample space according to the standard normal distribution and calculate all the corresponding response function values, and find out the total number of samples whose response value is less than zero, and the ratio of the total number to 200,000 is the required failure probability value. The comparison consequences are shown in Table 2.

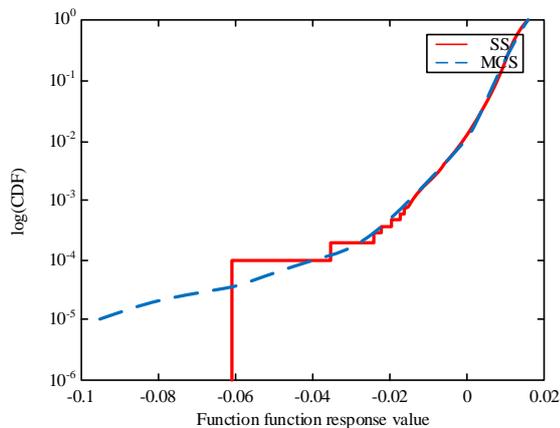
Table 2 Comparison of two methods

Analogy procedure	Failure probability	Total number of samples required
SS	$9.0 \times 10^{-3}$	5000
MCS	$9.5 \times 10^{-3}$	200000

Use MATLAB to draw the cumulative distribution curve of the response function. It is shown in Figure 2 and Figure 3.



**Figure 2** The CDF curve (a)



**Figure 3** The CDF curve (b)

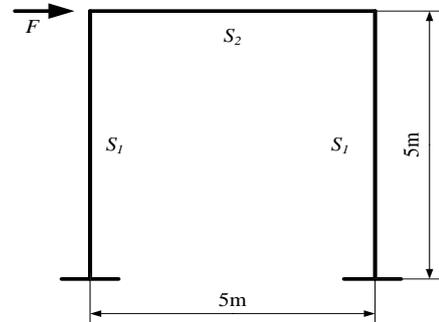
The results calculated by SS are represented by red real lines, and the results calculated by MCS are represented by blue virtual lines. It can be seen that the result of SS fits well with that of direct MCS, and it can be found from semi-logarithmic coordinate Figure 3 that the SS can also accurately fit the part with SFP. Only after the order of magnitude is  $10^{-4}$  will there be a significant difference. Therefore, it can be concluded that while ensuring a certain accuracy, the SS only needs 5000 samples to accurately calculate the failure probability of the failure mode corresponding to the response function, while the direct MCS needs dozens or even hundreds of samples. Therefore, the application of SS in SFP problem can greatly improve the computational efficiency and save computational resources.

#### 4.2 The example 2

Calculate the reliability of the single-layer portal frame structure as shown in the Figure 4. The elastic modulus values of beams and columns are both taken as:  $E = 2 \times 10^6 \text{ kN/m}^2$ . There is a correlation between the section moment of inertia and the section area:  $I_i = a_i S_i^2$ .  $F$  is the load variable,  $S_1$  and  $S_2$  respectively represent the cross-sectional area of the beam and column, and the statistical information of each random variable is shown

in Table 3. The response of the structure is taken as the displacement value of the top layer of the rigid frame, and the displacement limit of the structure is 0.01m, then the performance function of the structure based on displacement is:

$$Z = g(X) = 0.01 - u(X) \quad (22)$$



**Figure 4** The schematic diagram of single-layer portal frame

**Table 3** The information of random variables

Basic variables	Distribution type	Mean	Standard deviation	$\alpha_i$
A1	Lognormal	0.34	0.034	0.08234
A2	Lognormal	0.16	0.016	0.16333
P	Extreme value type I	20	5.0	--

In this section, the rod system model in the OpenSees software is used to simulate the plane rigid frame, and the beams and columns are taken as the basic elements.

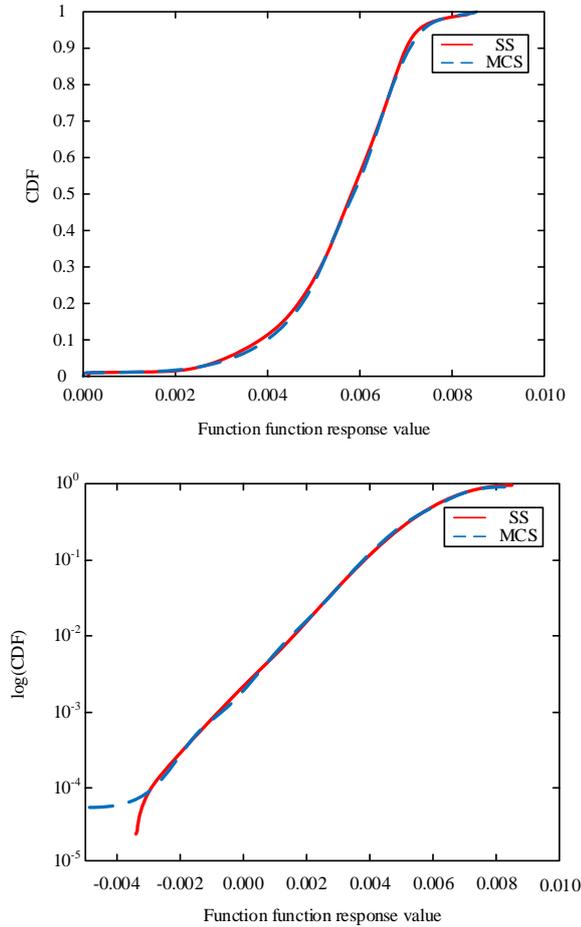
This section considers the correlation between A1 and A2. The nonlinear correlation coefficient  $\tau_k$  is set to 0.0 and 0.3, respectively, and the SS is used for reliability analysis. At the same time, the failure probability and reliability index obtained by MCS for 100,000 simulations are compared with the SS, as shown in Table 4.

From Table 4 can know that the SS greatly reduces the number of calculations and obtains more accurate and reliable indicators. As shown in Figure 5, through the cumulative distribution curve of the performance function, it can be seen that the curve fitting degree of the SS and the MCS method is better. In the semi-logarithmic coordinates, it can be seen that the cumulative distribution curve only has a significant difference after the order of magnitude is  $10^{-4}$ .

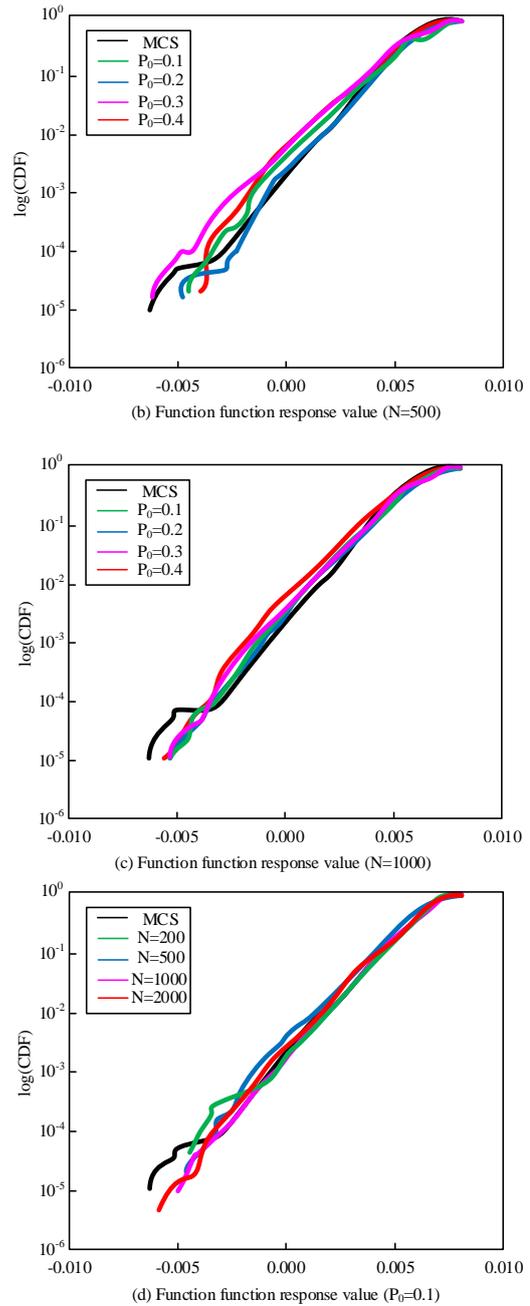
In the process of applying the SS, when the number of selected samples  $N$  and the conditional probability  $P_0$  are different, the calculated results are different. The number of samples is  $N=100, 200, 500, 1000$ ; the conditional probability is  $P_0=0.1, 0.2, 0.3, 0.4$ . Draw the cumulative distribution function curve, as shown in Figure. 6. The difference can almost be bridged with MCS for 100,000 times.

**Table 4** The result analysis and comparison

Correlation coefficient	Method	Reliable indicators	Probability of failure	Total sample points	Time (s)
τk=0.0	MCS	2.7910	2.130×10 <sup>-3</sup>	100000	14382
	SS	2.7994	2.060×10 <sup>-3</sup>	450	74
τk=0.3	MCS	3.0842	9.875×10 <sup>-4</sup>	100000	13985
	SS	3.0210	5.182×10 <sup>-4</sup>	500	91



**Figure 5** The CDF curve of performance function of portal frame



**Figure 6** The CDF curve of different initial sample points and failure probability combination

In Figure 6 (a), (b), (c), as the initial number of samples increases, the curve tail fitting becomes more accurate. It can be seen that the more the number of initial sample points, the better the simulation probability of a smaller range of points. When the initial sample points are small, the sample points with small occurrence probability will be ignored. For the selection of the conditional failure probability, when a smaller conditional probability is selected, the simulation curve of the SS and the curve of the MCS fit better, but the number of calculation subsets increases and the calculation efficiency decreases. It can be seen that the selection of

appropriate initial sample points and conditional failure probability has a certain impact on the accuracy and efficiency of the SS.

## 5 Conclusion

Aiming at the calculation of SFP in reliability analysis of engineering system, this paper discusses it based on SS. The SS adaptively extracts samples by applying MCMC method, and divides the whole probability space into a series of nested subspaces, which makes the subspaces approach to the failure area continuously. Therefore, the SFP of the target event can be accurately forecasted with a relatively small number of samples, and the calculation efficiency is improved. The reliability assessment method based on SS is constructed, and the analysis method is realized in two examples, which verifies the effectiveness of the proposed method. Then, this method is used in the engineering example to analyze the reliability problem.

## Acknowledgments

This research was funded by the Sichuan Science and Technology Innovation Seedling Project Funding Project (Grant No. 2021112).

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