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Multi-objective reliability optimization design of high-speed heavy-duty gears based on APCK-SORA model

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Abstract:
For high-speed heavy-duty gears in operation is prone to high tooth surface temperature rise and thus produce tooth surface gluing leading to transmission failure and other adverse effects, but in the gear optimization design and little consideration of thermal transmission errors and thermal resonance and other factors, while the conventional multi-objective optimization design methods are difficult to achieve the optimum of each objective. Based on this, the paper proposes a gear multi-objective reliability optimisation design method based on the APCK-SORA model. The PC-Kriging model and the adaptive k-means clustering method are combined to construct an adaptive reliability analysis method (APCK for short), which is then integrated with the SORA optimisation algorithm. The objective function is the lightweight of gear pair, the maximum overlap degree and the maximum anti-glue strength; the basic parameters of the gear and the sensitivity parameters affecting the thermal deformation and thermal resonance of the gear are used as design variables; the amount of thermal deformation and thermal resonance, as well as the contact strength of the tooth face and the bending strength of the tooth root are used as constraints; the optimisation results show that: the mass of the gear is reduced by 0.13kg, the degree of overlap is increased by 0.016 and the coefficient of safety against galling Compared with other methods, the proposed method is more efficient than the other methods in meeting the multi-objective reliability design requirements of lightweighting, ensuring smoothness and anti-galling capability of high-speed heavy-duty gears.
Keywords: *APCK-SORA model; high-speed heavy-duty gears; multi-objective reliability optimization design; k-means clustering method*

1 Introduction

High-speed and heavy-duty gears are widely used in aerospace and aviation, the marine industry and high-speed trains, and gear devices of various industries are also developing in the direction of high speed, heavy load and light weight. The surrogate model technique converts the actual complex structural problem into an approximate mathematical problem to be solved, which not only improves the computational efficiency of the optimised design model, but also allows the performance of the whole structure in the design space. Omar D. M et al [1] can change the contact pattern of tooth surfaces and proposed a structured optimisation method. Li et al [2] proposed a multi-objective ant colony optimisation model for improving the meshing performance and dynamic characteristics of gear transmission systems for high-speed heavy-duty herringbone gears used in the marine sector. Zhao et al [3] used the potential energy method to study the effect of tooth root cracking on gear meshing stiffness. Daniel et al [4] used a genetic algorithm to optimise the parameters of normal load, sliding speed and friction coefficient of a gear pair with multiple objectives and carried out an analytical calculation of the transmission efficiency of the gear. Dixit et al [5] used CRITIC (Criteria Importance through Intercriteria Correlation) method and Genetic Algorithm (GA) to obtain the optimal solution for multi-objective optimization considering the weight of the gear pair, power loss and gear heat treatment time. Maruti et al. [6] used an improved non-dominated sorting genetic algorithm (NSGA-II) to perform multi-objective optimization of three different gear profiles (unmodified profiles, smooth engagement profiles and high load-bearing energy profiles) and four ISO oil grades at two speeds. Edmund et al [7] carried out a multi-objective optimal design of a two-stage straight cylindrical gearbox with volume, power output and centre distance as objectives. Emna et al [8] considered both micro and macro parameters of gears, and used genetic algorithm to optimize gears for multi-objective optimization. Ekansh et al [9] carried out a multi-objective optimization of gears considering production cost, gear strength and noise impact parameters. Zhang [10] proposed a new collaborative strategy (C-RBMDO), which is a combination of a performance metric approach (PMA) and a parallel subspace optimization strategy (CSSO) and decouples the SORA for multidisciplinary optimal design of gears. At present, the sequence optimisation and reliability assessment methods are mainly based on the primary second order moment theory method for

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reliability analysis calculation, which has limited computational efficiency and accuracy. Some scholars have used different reliability methods in combination with SORA to make up for their shortcomings. For example, Cho and Lee [11] used a convex programming approach to transform the reliability optimisation column into a series of subplanning columns in a convex design domain, and introduced the hybrid mean value method (Hybird Mean Value,) to improve the computational performance of the SORA method. Du et al. [12] proposed a new search format for MPTP points to improve the robustness of the SORA method.On this basis, Cheng et al. [13] used the change of Angle in the iteration process to determine the convergence performance of MPTP search, and proposed to reduce the calculation times of non-tight constraint reliability information and improve the calculation efficiency of SORA by using the feasibility determination method of probability constraints. Ilchi [14] used sequence optimization and reliability analysis (SORA) to reduce the RBDO based on an improved adaptive chaos control method computational cost, a two-step improved adaptive chaos control method (DS-MACC) was proposed to speed up the cycle of reliability analysis. Subsequently, Ilchi [15] proposed a sequential optimisation and reliability analysis (SORA) method based on a PMA method for selecting adaptive step sizes at normalised locations and negative gradient vectors at two consecutive iteration points. Kaveh A [16] used sequential optimization with reliability assessment (SORA) as a decoupling method and proposed a reliability design optimization (RBDO) framework based on a meta-heuristic algorithm for decoupling methods. Kaveh A[17] used reliability-based design optimization (RBDO) to deal with these uncertainties and proposed to apply the sequential optimization with reliability assessment-dual meta-heuristic (SORA-DM) framework applied to RBDO of frame structures.

When high speed and heavy loads are applied to gears, the relative sliding between the tooth surfaces creates a lot of frictional heat, which raises the temperature of the gear teeth and alters the thermal expansion characteristics of the gear material. This leads to changes in the theoretical involute of the tooth profile and heat transfer errors of the gear, which have a negative impact on the accuracy, smoothness, and noise level of the transmission. In order to solve such problems, an adaptive surrrogate model-based reliability optimisation method based on an improved SORA optimisation algorithm with an adaptive PC-Kriging model is proposed. Firstly, a new adaptive structural reliability analysis method (referred to as APC-Kriging) is constructed by combining the PC-Kriging model with an adaptive k-means clustering method. Secondly, the proposed adaptive PC-Kriging model is used to solve the reliability part of the SORA optimisation algorithm and then to optimise the design by SORA. In order to achieve a multiobjective optimised design for the reliability of high speed heavy load gears with lighter volume, better transmission smoothness and anti-galling capability.

2 High-speed heavy-duty gear model considering the effect of temperature rise

To improve the transmission accuracy, smoothness and load carrying capacity of high-speed heavy-duty gears, a finite element analysis of gears considering temperature rise is carried out, i.e. a thermal analysis of gears based on thermal-structural coupling. For the thermal analysis, Solid70 three-dimensional solid units are used, and the material properties such as heat transfer coefficient, heat flow density, modulus of elasticity, specific heat capacity and related boundary conditions are set. The model is divided by sweeping the mesh, and only the mesh refinement of its mesh surface, while the rest of the mesh density can be relatively sparse, divided into the master gear and driven gear of the finite element model as shown in Figure 1, the relevant loading coefficients are shown in Table 1.

Table 1 Loading factors

Modulus of elasticity	Poisson's ratio	Thermal conductivity	Heat flow density	Linear expansion coefficient
206	0.3	29.7	542.2	10.36

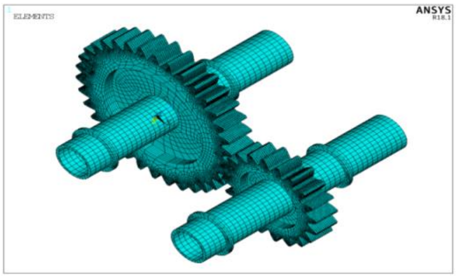
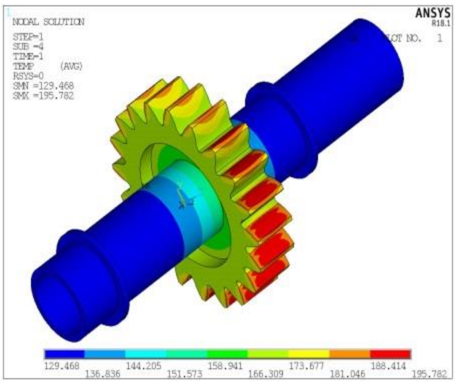
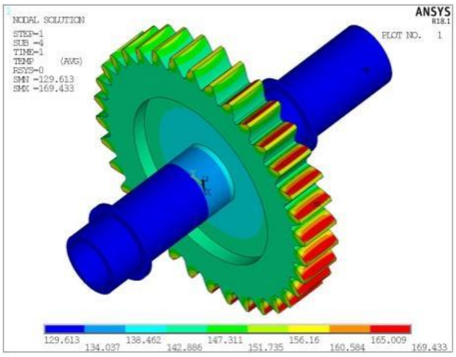


Figure 1 The three-dimensional solid model of the gear pair after meshing

According to the literature [18], the heat flow density and convective heat transfer coefficients are calculated for each face of the gear and are applied simultaneously to the meshing face of the gear, and the convective heat transfer coefficients are applied to the non-meshing, top and root faces of the gear as well as the end face of the gear with relevant settings, and finally the steady-state temperature field of the gear is solved. The steady-state temperature field clouds of the driven and driven gears are shown in Figure 2(a) and (b).



Cloud of active wheel temperature field distribution



Cloud of driven wheel temperature field distribution

Figure 2 Steady Temperature Distribution of gear

As can be seen from Figure 2, the maximum temperature on the meshing tooth surface of the active gear is (195.702 ℃) and the maximum temperature on the meshing tooth surface of the driven gear is (169.433 ℃), and the high temperature areas on the meshing tooth surface of both the active and driven gears are found near the root and the top of the teeth, which is due to the fact that the contact compressive stress between the teeth and the relative sliding speed between the teeth during the transmission process of the gears is greater in these two areas. Because in the process of gear transmission, the product of the contact compressive stress between the tooth surfaces and the relative sliding speed between the teeth is large at these two places, and then more friction heat input is generated, which is consistent with the fact that gear gluing usually occurs near the root of the driving wheel or near the top of the driven wheel. It can be proved that the analysis method of gear steady-state temperature field is correct and effective.

3 Mathematical model of APCK-SORA

An adaptive structural reliability analysis method based on a combination of PC-Kriging and adaptive k-means (referred to as APC-Kriging) is proposed. Firstly, PC-Kriging is an improved Kriging algorithm whose regression basis function uses a sparse polynomial optimal truncation set to approximate the global behaviour of the numerical model, and Kriging is used to handle local variations in the model output, which improves the computational efficiency while ensuring accuracy. Secondly, while common reliability methods collect sample points one by one, the adaptive k-means clustering analysis in this paper divides the space into several regions and selects an optimal sample point from each region, thus enabling multiple regions to simultaneously achieve the aim of improving the accuracy of the PC-Kriging model, thus again improving the computational efficiency of the model. Finally, the proposed adaptive PC-Kriging model is combined with SORA to construct a multi-objective reliability optimisation method based on the adaptive surrogate model.

3.1Adaptive PC-Kriging reliability model

The adaptive PC-Kriging method uses a k-means cluster analysis approach to ensure that a number of sample points that contribute significantly to the probability of failure are added at each iteration. The main steps of the proposed adaptive PC-Kriging method for selecting sample points are as follows:

Step 1 t=0. The initial experimental design sample points are generated by random sampling of the Latin hypercube and the corresponding functional response values are computed exactly, i.e. $\hat{G}_0(\boldsymbol{x})$, $\hat{P}_{f,0}$. Let the initial number of sample points be M_0 , then we have $\Omega_0=\{(x_{0,i},y_{0,i}),i=1,2,...,M_0\}$, $X_0=\left\{x_{0,1},x_{0,2},\dots,x_{0,M_0}\right\}$.

Step 2 t=t+1, generating K points by Markov chain Monte Carlo simulation (MCMC) on $\hat{G}_{t-1}(\boldsymbol{x})=0$. Given $\hat{G}_{t-1}(\boldsymbol{x})$, a random vector of M dimensions obeying f(x) (f(x) is the joint probability density function of x) and satisfying equation (1) is generated using the MCMC method, then the randomly selected points are considered to be on $\hat{G}_{t-1}(\boldsymbol{x})=0$. The random extraction process stops when the number of extracted points reaches K. The random vector generated is $\tilde{\boldsymbol{X}}_{t-1}=\{\tilde{\boldsymbol{x}}_{t-1,1},\tilde{\boldsymbol{x}}_{t-1,2},...,\tilde{\boldsymbol{x}}_{t-1,K}\}$, and in this paper, let K be 2000 and [ε] be 0.01.

$$|\hat{G}_{t-1}(\boldsymbol{x})|\leq[\varepsilon] \tag{1}$$

Step 3 The k-means cluster analysis method is used to divide $\tilde{\boldsymbol{X}}_{t-1}$ into k categories and map the centroids of these k categories onto $\hat{G}_{t-1}(\boldsymbol{x})=0$. Let $\left\{s_{t-1,1},s_{t-1,2},\dots,s_{t-1,k}\right\}$ denote the k clustering centres. When $\hat{G}_{t-1}(\boldsymbol{x})=0$ is a non-linear surface, it is not guaranteed that the centroids of the k categories are all on $\hat{G}_{t-1}(\boldsymbol{x})=0$, and it is necessary to map the centroids that are not on $\hat{G}_{t-1}(\boldsymbol{x})=0$ to $\hat{G}_{t-1}(\boldsymbol{x})=0$. The mapping is done by finding the points that satisfy equation (2) and obtaining $S_{t-1}=\{\hat{\boldsymbol{s}}_{t-1,0},\hat{\boldsymbol{s}}_{t-1,1},...,\hat{\boldsymbol{s}}_{t-1,k}\}$, where $\hat{\boldsymbol{s}}_{t-1,0}$ is the design point for $\hat{G}_{t-1}(\boldsymbol{x})$.

$$\left.\begin{array}{r} min\ \|\boldsymbol{x}-\boldsymbol{s}_{t-1,i}\| \\ s.t.\hat{G}(\boldsymbol{x})=0 \end{array}\right\} \tag{2}$$

where i=1,2,...,k.

Step 4 Adjust the positions of the points in the set S_{t-1}. Define the distance D_0 as shown in equation (3) and assume that if the distance between any two sample points in the set S_{t-1} is less than D_0 it is considered unacceptable and the location of individual points in the set S_{t-1} needs to be adjusted.

$$D_0=e\cdot\left(\frac{2}{M(M-1)}\sum_{i<j}\|\boldsymbol{x}_i-\boldsymbol{x}_j\|\right) \tag{3}$$

where e is a given constant.

There are two possible scenarios for the points in the set S_{t-1}: 1. the distance between some points within S_{t-1} may be too small. $\hat{\boldsymbol{s}}_{t-1,0}$ It is more likely that the distance to some point in $\hat{\boldsymbol{s}}_{t-1,1},...,\hat{\boldsymbol{s}}_{t-1,k}$ is small; 2. The distance between some point in S_{t-1} and some point in X_{t-1} is too small. If case 1 occurs, e.g. the distance between $\hat{\boldsymbol{s}}_{t-1,1}$ and $\hat{\boldsymbol{s}}_{t-1,2}$ is less than D_0 , the position of the point with the smaller probability density function is to be changed and the position of the point with the larger probability density function is to be left unchanged; if case 2 occurs, the corresponding point in S_{t-1} is to be changed. The sample points are adjusted by assuming that the position of $\hat{\boldsymbol{s}}_{t-1,1}$ is to be changed first. The points in $\tilde{\boldsymbol{X}}_{t-1}$ are arranged in ascending order of distance from $\hat{\boldsymbol{s}}_{t-1,1}$ and $\hat{\boldsymbol{s}}_{t-1,1}$ is changed to the new sequence of points in turn until the distance between $\hat{\boldsymbol{s}}_{t-1,1}$ and all points in S_{t-1} and X_{t-1} is greater than D_0 .

Step 5 Calculate the value of the function corresponding to each sample point in the set S.

$$\Omega_{t-1}^0 = \{(\hat{\boldsymbol{s}}_{t-1,i}, y_{t-1,i}), i = 0,1,...,k\} \text{ , } \Omega_t = \Omega_{t-1} \cup \Omega_{t-1}^0 \quad (4)$$

Step 6 Calculate $\hat{G}(\boldsymbol{x})$, $P_{f,t}$ based on Ω_t and combining Eqs. (1) and (2). If the convergence condition of equation (4) is satisfied, the iterative process stops and $\tilde{P}_{f,t}$ is the estimated value of P_f ; otherwise, return to step 2 until $\tilde{P}_{f,t}$ satisfies the convergence condition.

$$\frac{N_{un}}{N_{fail}} \leq \alpha \quad (5)$$

where N_{un} is the total number of samples with sign prediction errors, N_{fail} represents the total number of failed samples, and α is the permissible error of $\hat{P}_f$, where

$$N_{un} = 2\left(\frac{N_{U<P}}{2} + N_{P\leq U\leq Q} \cdot \Phi(-U)\right) \quad (6)$$

$$N_{fail} = \sum_{i=1}^{N_{MC}} I_{\hat{G}}(\boldsymbol{x}_i) \quad (7)$$

In this case, sample points with a high probability of symbol prediction error are indicated by $N_{U<P}$, and such points can be considered as certain failure points. Sample points with a high probability of symbol prediction error are indicated by $N_{P\leq U\leq Q}$, and the total number of failure prediction errors for such sample points can be indicated by $N_{P\leq U\leq Q} \cdot \Phi(-U)$. In this study, P=1, Q=2 and α=0.03.

3.2 Development of the APCK-SORA mathematical model

Firstly, PC-Kriging is used to approximate the global behaviour of the numerical model, and Kriging is used to deal with local variations in the model output. Secondly, adaptive k-means cluster analysis is used to divide the space into several regions and select an optimal sample point from each region, so that multiple regions can simultaneously achieve the objective of improving the accuracy and computational efficiency of the PC-Kriging model. Finally, in combination with the SORA optimisation strategy (i.e. separating the reliability assessment from the optimisation design), the model converges and obtains an optimal solution with a small number of cycles, making the solution of complex optimisation design problems simple and efficient.

The mathematical model for APCK-SORA based reliability optimization is:

$$\begin{aligned}
&\text{DV} = (\boldsymbol{d}, \boldsymbol{X}^M) \\
&\min \quad f(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M) = \{f_1(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M), f_2(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M), \cdots, f_m(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M)\} \\
&s.t. \quad G_i(\boldsymbol{d}, \boldsymbol{X}^M - \boldsymbol{s}_i, \boldsymbol{Y}_{MPPi}) \geq 0, i = 1,2,\cdots,n \\
&\qquad g_j(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M) \leq 0, j = 1,2,\cdots,p \\
&\qquad h_k(\boldsymbol{d}, \boldsymbol{X}^M, \boldsymbol{Y}^M) = 0, k = 1,2,\cdots,q \\
&\qquad \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U \\
&\qquad \boldsymbol{X}^{M,L} \leq \boldsymbol{X}^M \leq \boldsymbol{X}^{M,U}
\end{aligned} \quad (8)$$

where $\boldsymbol{s}_i = \boldsymbol{X}^{M(k-1)} - \boldsymbol{X}_{MPPi}^{(k-1)}$

The reliability analysis section is then solved using the APC-Kriging proposed in section 3.1. It is determined whether the reliability requirements are met and if not, a deterministic optimisation model is constructed for the next cycle.

3.3 Solving steps for APCK-SORA

In each cycle of SORA, deterministic optimisation is first carried out, followed by reliability analysis. the basic flow of the APCK-SORA method (Figure 4) and the main steps are shown below:

(1) The initial experimental design sample points were first generated by random sampling of the Latin hypercube, and the corresponding functional response values were calculated exactly, i.e. $\hat{G}_0(\boldsymbol{x})$, $\tilde{P}_{f,0}$. Let the initial number of sample points be M_0 .

(2) Solve for deterministic optimization. Set the initial value of the optimisation design variable to $d^{(0)}, \boldsymbol{X}^{M(0)}$ and the superscript 0 to indicate that no reliability analysis has been performed. Starting from the 1st cycle, get $\boldsymbol{X}_{\text{MPP}i}^{(k)}$ and $\boldsymbol{s}_i$ in the deterministic optimization to build a completely new optimization calculation model.

(3) Perform a reliability analysis at the optimal design points $\boldsymbol{d}^{(k)}$, $\boldsymbol{X}^{M(k)}$ obtained in the deterministic optimization and find the corresponding $\boldsymbol{X}_{\text{MPP}i}^{(k)}$.

(4) Test for feasibility and convergence. If all reliability constraints and deterministic constraints are satisfied and the system objective function values converge, $\left\| f^{(k)} - f^{(k-1)} \right\| \leq \varepsilon$, ε to a 0.001, then the reliability optimization process stops. Instead, calculate $\boldsymbol{s}_i$ based on the current MPP, adjust the position of the design variable $\boldsymbol{X}^M$ to ensure that the constraint boundaries are within the feasible domain, and go to step (3).

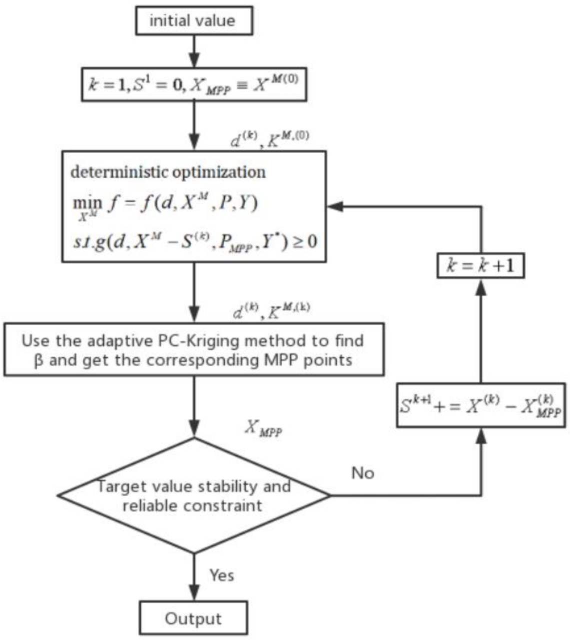


Figure 3 The flow diagram of APCK-SORA

4 Multi-objective reliability optimisation of high-speed heavy-duty gears design

4.1 Objective function

(1) Minimum sum of masses of gear pairs

Considering the involute straight cylindrical gear as a cylinder, the diameter as its index circle diameter and the height as the tooth width, the total mass of the drive system is:

$$M = M_1 + M_2 = \frac{\pi}{4}(d_1^2 b_2 + d_2^2 b_2)\rho \tag{9}$$

(2) Maximum overlap

In order to ensure the continuity of the gearing, the degree of overlap ε must be greater than or at least equal to 1. The greater the value of ε, the better the continuity of the gearing and the smoother the transmission.

$$\varepsilon = \frac{1}{2\pi}\left[z_1(\tan\alpha_1 - \tan\alpha) + z_2(\tan\alpha_2 - \tan\alpha)\right] \tag{10}$$

Where $\alpha_1 = \arccos\frac{d_{b1}}{d_{a1}}$, $\alpha_2 = \arccos\frac{d_{b2}}{d_{a2}}$, d_{a1}, d_{a2} are the top circle diameters and d_{b1}, d_{b2} are the base circle diameters.

(3) Maximum resistance to gluing

The instantaneous contact criterion (Blok flash temperature method) considers that the hot glue damage is caused by the high temperature generated by friction at the contact point, which ruptures the lubricant film and causes a sharp increase in the coefficient of friction, resulting in a higher temperature and the formation of a sticky weld between the metals, which tears open the weld joint due to the relative movement and thus forms the glue damage. In engineering, the Blok flash temperature method of hot bonding strength conditions are:

$$t_{C\max} = t_M + T_{t\max} \le t_s \tag{11}$$

The factor of safety for gear gluing strength is given by Eq:

$$S_B = \frac{t_s - t_{oil}}{t_{C\max} - t_{oil}} \tag{12}$$

Where: $t_{C\max}$ is the maximum contact temperature on the engagement surface, ℃; t_M is the body temperature, ℃; $T_{t\max}$ is the maximum flash temperature on the contact surface, ℃; t_s is the critical gelling temperature, ℃; t_{oil} is the lubricant temperature at thermal steady state, ℃. Since the limiting temperature of the lubricant is 220 ℃, the critical bonding temperature is set to 220 ℃ in this paper. Where $t_{C\max}$ and t_{oil} are obtained through finite element simulation, the greater the safety factor of gear gluing strength S_B , the greater the resistance to gluing, which means that the maximum contact temperature of the meshing surface $t_{C\max}$ should be lower.

4.2 Design variables

The number of teeth, modulus and tooth width of the active and driven gears are selected as the optimisation design variables, while the other basic design parameters of the gearing system remain unchanged:

$$\begin{aligned} x &= [x_1, x_2, x_3, x_4, x_5]^{\mathrm{T}} \\ &= [m, z_1, b_1, z_2, b_2]^{\mathrm{T}} \end{aligned} \tag{13}$$

4.3 Constraints

(1) Heat deflection constraint

In order to avoid the thermal deformation of high speed heavy duty gears leading to "jamming" of the gearing, it is necessary to ensure that the thermal deformation of the gear is less than the minimum side clearance of the gear (the amount by which the width of the tooth groove on the pitch line is greater than the tooth thickness).

$$j_{\min} = \frac{2}{3}(0.06 + 0.0005a + 0.03m) \tag{14}$$

$$G_1(X) = j_{\min} - \Delta\delta_{\max} \ge 0 \tag{15}$$

where $J_{\min}$ is the minimum side clearance of the gear, a is the centre distance and $\Delta\delta_{\max}$ is the maximum tooth deflection from the ANSYS simulation.

(2) Thermal resonance frequency constraint

According to the resonance principle, the gear rotor will resonate when the excitation frequency is close to or equal to the intrinsic frequency. According to reliability disturbance theory, the state function for random structural failure analysis is:

$$G_{2,i}(X) = |p - \omega_i|, (i = 1, 2 \ldots n) \tag{16}$$

Geared rotor systems constructed to avoid resonance

$$G_{2,i}(X) = |p - \omega_i| - \gamma > 0 \tag{17}$$

where p is the excitation frequency; ω_i is the ith intrinsic frequency, γ is 10% of the intrinsic frequency of each order of the gear rotor.

(3) Gear ratio constraint

The transmission ratio is the ratio of the angular velocities of the two involute gears. The optimised design (lightweighting) will result in a change in the number of teeth of the gear pair, which needs to be constrained in order not to affect the transmission ratio of the gear pair.

$$i_{12} = \frac{z_2}{z_1} \tag{18}$$

$$G_3(X) = 0.05 - |i_{12} - 1.7| / 1.7 > 0 \tag{19}$$

Where, i_{12} is the ratio between the active and driven gears, z_1 and z_2 are the number of teeth of the active and driven gears respectively.

(4) Centre distance constraint

The property that the centre distance between the gear pairs changes while the two ratios remain unchanged is known as the divisibility of involute gears. However, considering the overall dimensions of the gearbox, the centre distance of the gear pairs is therefore constrained.

$$\nabla a = |a - a_0| \tag{20}$$

$$G_4(X) = \nabla a = 0 \tag{21}$$

Where ∇a is the change in the initial and optimised centre distance between the gear pairs, a_0 is the initial centre distance between the gear pairs and a is the optimised centre distance between the gear pairs.

(5) Variation factor constraint

In the optimised design of gears the number of teeth and modules may be improved, resulting in the theoretical centre distance being unequal to the actual centre distance, when it is often necessary to adjust the coefficient of variation to meet the centre distance constraint. Where the total coefficient of variation is:

$$x_{\Sigma} = \frac{(\mathrm{inv}\alpha' - \mathrm{inv}\alpha)\cdot(z_1+z_2)}{2tg\alpha} \tag{22}$$

$$\alpha' = \arccos\left(\frac{a}{a'}\cos\alpha\right) \tag{23}$$

Where, α is tooth angle, α' is engagement angle, a isideal centre distance, a' is actual centre distance.

For the assignment of the coefficient of variation, due to the high speed of high-speed heavy-duty gears, the coefficient of variation of involute straight cylindrical gears is assigned using the method of gluing failure[18] Constraint on it

$$G_5(X) = x_1,\ \text{where}\ \begin{cases} x_1 = \dfrac{x_{\Sigma}}{i+1} g \dfrac{i-1}{i+1+0.4z_2} \\ x_2 = x_{\Sigma} - x_1 \end{cases} \tag{24}$$

Where x_{Σ} is the total displacement factor, x_1 , x_2 are the displacement factors of the master and driven gears respectively, and i is the transmission ratio.

(6) Number of teeth constraint

Considering the minimum number of teeth required for gearing and the distribution of ratios, the number of teeth of the master and driven gears is taken as an integer, and the minimum number of teeth for a standard straight cylindrical gear without heel tangency is 17, so the range of values is as follows: $\begin{cases} 17 \le z_1 \le 24 \\ 30 \le z_2 \le 38 \end{cases}$, z_1, z_2 are integers

$G_6(X) = z_1 - 17 > 0$.

where $G_5(X)$ is the number of teeth of the main and driven gears respectively.

(7) Modulus constraint

Modulus m is a basic parameter of gears, the larger the modulus, the larger the tooth pitch of the gear, the modulus of standard straight cylindrical gears has been standardized, the modulus selection range is as follows:

m=(1.0,1.25,1.5,1.75,2.0,2.25,2.5,2.75,3)

Referring to the mechanical design manual take the modulus greater than or equal to 2.5, i.e: $G_7(X) = m - 2.5 > 0$.

(8) Tooth width constraint

Calculated from the tooth width of the large gear (driven wheel) $b_2 = \Phi_d * d_2$, which should be rounded off and used as the tooth width of the large gear. Required tooth width of the pinion (active wheel): $b_1 = b_2 + (0.5$ to $1.0)$ mm.

$$\Phi_d < 1.2 \tag{25}$$

$$G_8(X) = 1.2 - \Phi_d \ge 0 \tag{26}$$

Where Φ_d is the tooth width factor and d_1 is the pinion indexing circle diameter.

(9) Strength constraints for gears

Strength constraints for gears include gear contact strength constraints and tooth root bending strength constraints. The tooth contact strength is related to the tooth contact stress and the permissible contact stress.

The constraint function for the contact strength of the tooth surface σ_H is: $G_9(X) = \sigma_H - [\sigma_H] < 0$.

The root bending strength is related to the permissible bending stress of the gear material. The constraint functions for the root bending strengths σ_{F1} and σ_{F2} of the pinion and large gears respectively are:

$$\begin{aligned} G_{10}(X) &= \sigma_{F1} - [\sigma_F]_1 < 0 \\ G_{11}(X) &= \sigma_{F2} - [\sigma_F]_2 < 0 \end{aligned} \tag{27}$$

4.4 Reliability optimised design of gears

The mathematical model for reliability optimisation of thermally-structurally coupled gears is:

$$\begin{aligned} &\text{Design variables} \quad \boldsymbol{X}^M = (x_1^M, x_2^M, x_3^M, x_4^M, x_5^M) \\ &\min \quad f(\boldsymbol{X}^M) = \{f_1(\boldsymbol{X}^M), f_2(\boldsymbol{X}^M), f_2(\boldsymbol{X}^M)\} \\ &st. \quad G_i(\boldsymbol{X}^M\text{-}s) \ge 0 \\ &\quad f_1(\boldsymbol{X}^M) \ge 0 \\ &\quad f_2(\boldsymbol{X}^M) \ge 1.4 \\ &\quad f_3(\boldsymbol{X}^M) \le S_B \\ &\quad 2.5 \le x_1^M < 4 \\ &\quad 17 \le x_2^M \le 24 \\ &\quad 30 \le x_3^M \le 38 \\ &\quad 13 \le x_4^M < 16 \\ &\quad 13 \le x_5^M < 16 \end{aligned} \tag{28}$$

where $\boldsymbol{s}_i = \boldsymbol{X}^{M(k-1)} - \boldsymbol{X}_{\mathrm{MPP}}^{(k-1)}$, $\boldsymbol{X}^{M(k-1)}$ and $\boldsymbol{X}_{\mathrm{MPP}i}^{(k-1)}$ are the deterministic optimisation solution and the most probable point (MPP) obtained from the previous loop, respectively. A reliability analysis is performed at the most probable point to calculate the MPP point in the probability constraint $(\boldsymbol{X}_{\mathrm{MPP}}^k, \boldsymbol{Y}_{\mathrm{MPP}}^k)$ and the shift vector for the next loop constraint $\boldsymbol{S}^{k+1}$, which in turn constructs the deterministic optimisation model for the next iteration.

The specific steps for the optimal reliability design of thermally-structurally coupled gears are as follows:

(1) The objective function is first defined.

$$f_1(\boldsymbol{X}) = M(\boldsymbol{X}) = M_1 + M_2 = \frac{\pi}{4}(d_1^2 b_2 + d_2^2 b_2)\rho \tag{29}$$

$$f_2(\boldsymbol{X}) = \varepsilon = \frac{1}{2\pi}\left[z_1(\tan\alpha_1 - \tan\alpha) + z_2(\tan\alpha_2 - \tan\alpha)\right] \tag{30}$$

$$f_3(\boldsymbol{X}) = S_B = \frac{t_s - t_{oil}}{t_{C\max} - t_{oil}} \tag{31}$$

The objective is to minimise the mass of $f_1(\boldsymbol{X})$, maximise the degree of overlap $f_2(\boldsymbol{X})$ and maximise the coefficient of safety against gluing $f_3(\boldsymbol{X})$ according to the design requirements and to satisfy the constraints.

Considering the 3 design requirements simultaneously, uses a linear weighted combination method to convert the 3 sub-objective functions into a single objective optimization function $F(\boldsymbol{X})$, such that

$$F(\boldsymbol{X}) = \lambda_1 f_1(\boldsymbol{X}) + \lambda_2 f_2(\boldsymbol{X}) + \lambda_3 f_3(\boldsymbol{X}) \tag{32}$$

where λ_i (i=1,2,3) denotes the weighting factors , λ_1=0.3, $\lambda_2=-0.3$, $\lambda_3=-0.4$ and therefore $\min F(\boldsymbol{X})$ as the final optimisation target.

(2) The initial sampling in the design space is carried out using the Latin square design of experiments method and the corresponding response values are calculated at $F(\boldsymbol{X})$. For $f_1(\boldsymbol{X})$ and $f_2(\boldsymbol{X})$, the equations are calculated and for $f_3(\boldsymbol{X})$ $t_{C\max}$ is used to obtain the maximum contact temperature of the meshing surfaces by a MATLAB call to ANSYS software for coupled thermal-structural analysis of the gear.

(3) Construct PC-Kriging approximation models for the objective and constraint functions. The deterministic optimal design is then based on the current approximate model to obtain the current optimal solution. At the initial first iteration step, set $\boldsymbol{u}_{MPTP} = \boldsymbol{0}$.

(4) Given μ_{X}, σ_X, a reliability analysis is performed by the adaptive PC-Kriging reliability method to obtain the $\boldsymbol{u}_{MPP}$ point. The optimised solution is transformed from the design space into the coordinate space where the random variable X is located to obtain its response value i.e. the current minimum value G_{min} If more than one constraint exists, each constraint function will obtain a minimum value $G_{i,min}$ (i=1,...,N).

(5) The best sample points are selected using the adaptive PC-Kriging method and added to the initial sample to reconstruct the PC-Kriging model.

(6) Update all sample points to construct the PC-Kriging approximation model and find all constrained MPP points. If at this point all $\hat{G}_{i,MPP} \geq 0, i = 1,..., N$, and the objective function value does not change much, then stop the iterative process that is the final result; otherwise if there is any $\hat{G}_{i,MPP} \geq 0$, otherwise, $k = k + 1$, then based on the current result return to step (4) to re-optimize the solution.

The final optimisation results obtained through two iterations of the optimisation design are shown in Table 2.

As can be seen from Table 2, the adaptive agent model's optimised design approach (APCK-SORA) is used to optimise the reliability design of high-speed heavy-duty gears. Firstly, a feasible domain of gear modulus, number of teeth and tooth width satisfying the conditions is selected based on the constraints of gear modulus, number of teeth, transmission ratio and strength. On the basis of this, optimisation is carried out for the objective functions of minimising the sum of the masses of the gear pairs, maximising the degree of overlap and maximising the resistance to galling. The optimisation results are shown in Table 2. It can be seen that the number of teeth of the master and driven wheels has been increased, but the modulus has been reduced from 3 mm to 2.75 mm, and the width of the master and driven wheels has been reduced. In order to meet the centre distance constraint, the optimised master and driven gears were machined using a positive displacement method. The result of the optimisation is a reduction in mass of 0.13kg, an increase in the degree of overlap of 0.016 and an increase in the coefficient of safety against galling of 0.19. This achieves the optimised design objectives of light weight, smoothness of transmission and maximum reliability against galling.

Table 2 Comparison of results before and after optimization

Reference items	Initial value	Results of one iteration	Optimal results
m(mm)	3	2.75	2.75
z_1	20	20	22
z_2	34	34	36
b_1 (mm)	15	14.7	14.0
b_2 (mm)	14.5	13.9	13.5
Mass (kg)	1.26	1.15	1.13
Overlap	1.618	1.628	1.634
Anti-glueing safety factor	1.24	1.38	1.43

Table 3 Comparison of different optimization results

Reference items	SAP with PMA	SORA	SLA	The proposed method
Number of iterations	308+5	296+2	265+4	53+2
m(mm)	2.75	2.75	2.75	2.75
z1	20	20	20	20
z2	34	34	34	34
b1 (mm)	14.8	14.7	14.8	14.7
b2 (mm)	14.0	13.9	13.8	13.9
Mass (kg)	1.16	1.15	1.15	1.15
Overlap	1.628	1.628	1.628	1.628
Anti-glueing safety factor	1.39	1.38	1.37	1.38

The first row of data in Table 3 is 308/5, which means that the number of optimisation iterations is 5 and the number of iterations to build the maximum contact temperature model for the meshing surface is 308. It can be seen from the comparison that the optimisation results obtained using the four algorithms are basically the same, but the proposed method only requires 53 iterations to complete the construction of the maximum contact temperature model for the meshing surface, and the parameters are optimised by two optimisation iterations, which has a higher computational efficiency.

5 Conclusion

An adaptive PC-Kriging model is proposed to improve the reliability part of the SORA optimization algorithm for multi-objective reliability optimization design of high-speed heavy-duty gears with an adaptive agent model. The objective functions of minimizing the sum of gear pair masses (lightweighting), maximizing degree of overlap (ensuring smooth transmission), and maximizing strength against galling are utilized to achieve multi-objective reliability optimization design for high-speed heavy-duty gears.

Comparing the proposed method with other optimization algorithms, it can be seen that while the final optimization results are essentially the same, the overall efficiency of modeling and optimization has been greatly improved. The proposed method has significant engineering value and is particularly well-suited for addressing reliability optimization problems in practical engineering applications, which typically require substantial investments of time and resources to construct an optimal model.

High-speed heavy-duty gears are inevitably affected by a variety of random factors during operation, such as fluctuations in external loads, changes in the environment and changes in the thermophysical properties of gear materials. Future research should consider the multi-objective optimization of gear design for dynamic reliability of relevant parameters over time.

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